CMB COMPUTATIONAL COOKBOOK

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CMB MEETING
MONTREAL, MARCH 2008

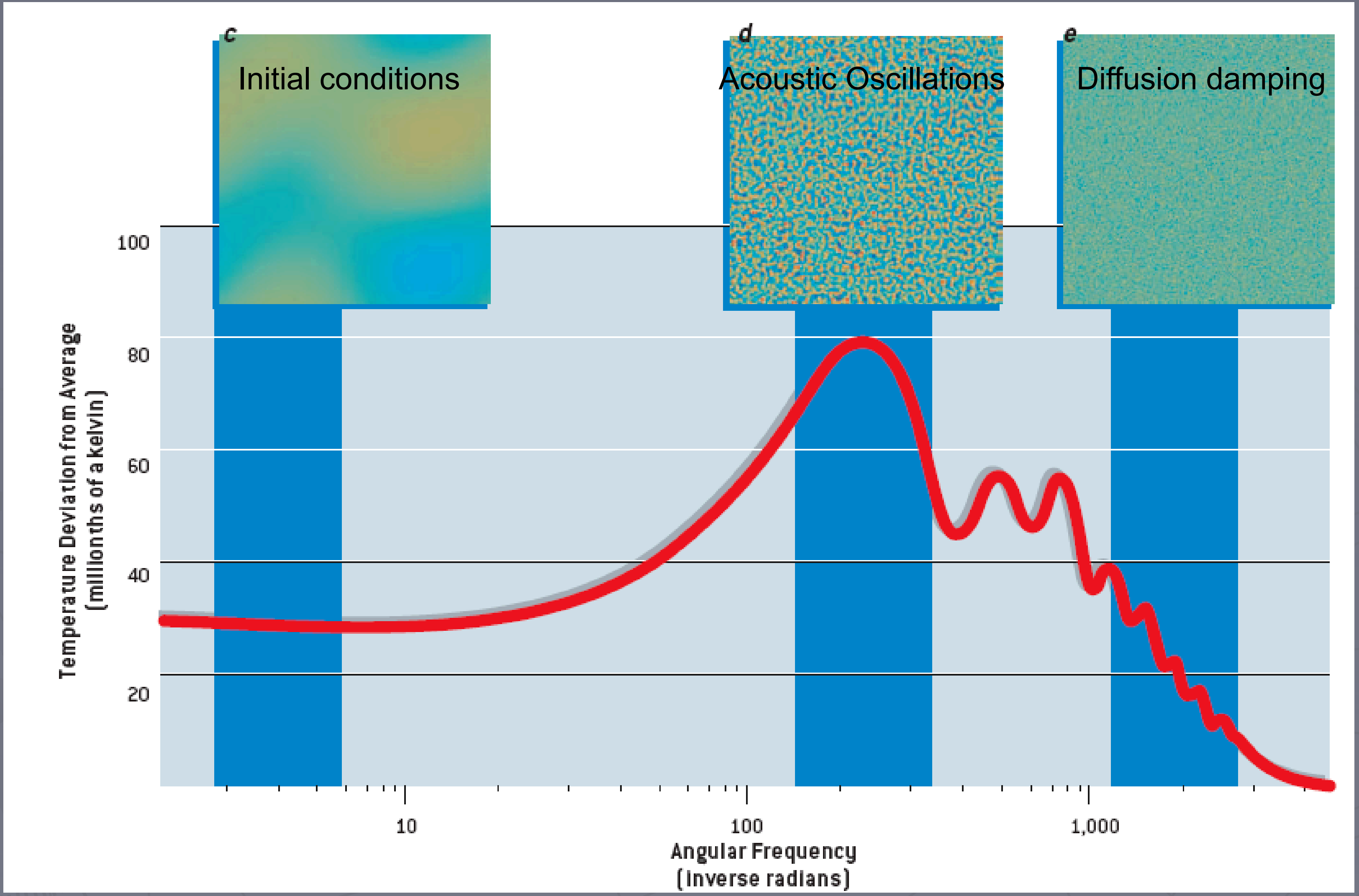
Motivation/Plan of talk

- Precision cosmology requires precision tools
  - CMB codes compute power spectrum accurate < 1 %
  - Codes are 1000's lines - difficult to decompose
  - Black box - need understanding of intermediate steps!
- Plan of talk:
  - Basic physics to understand complex code
  - Demonstration of code to gain intuition
  - Breakdown of a CMB code
  - Examples of how to modify code to include new physics

Resources

- CMB codes
  - CMBFAST (cmbfast.org)
  - CAMB (camb.info)
  - CMBEASY (cmbeasy.org)
- Recombination physics
  - RECFAST (astro.ubc.ca/people/scott/recfast.html)
- Parameter estimation
  - COSMOMC (cosmologist.info/cosmomc)
  - COSMONet (mrao.cam.ac.uk/software/cosmonet)
- Sky pixelization and visualization
  - HEALPix (healpix.jpl.nasa.gov)

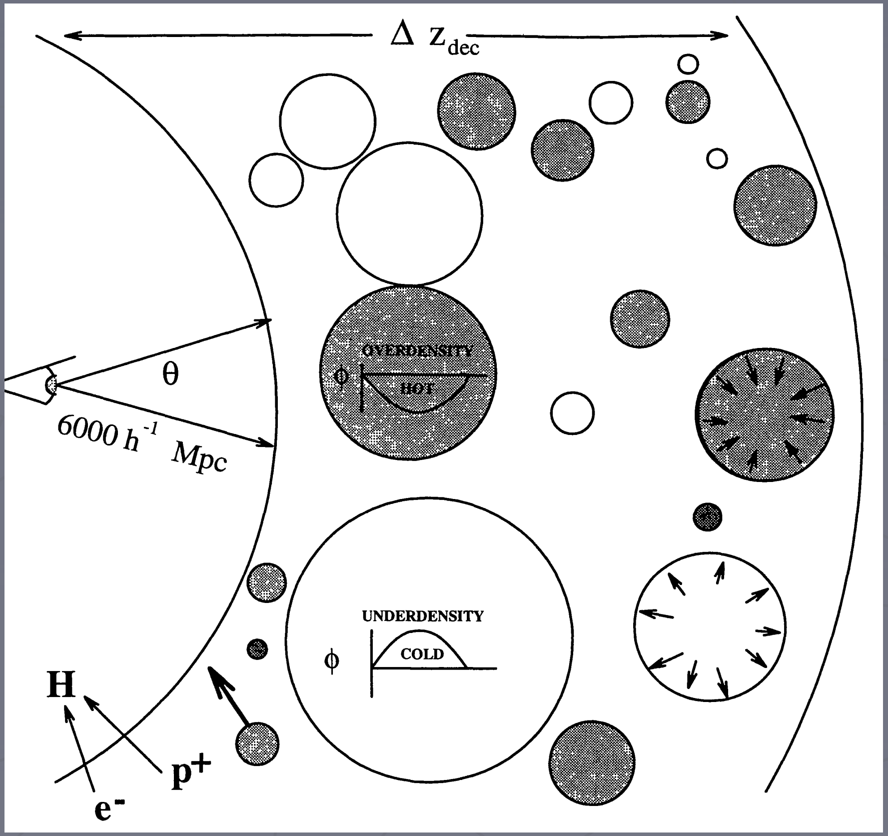
CMB power spectrum



(Hu and White, 2004)

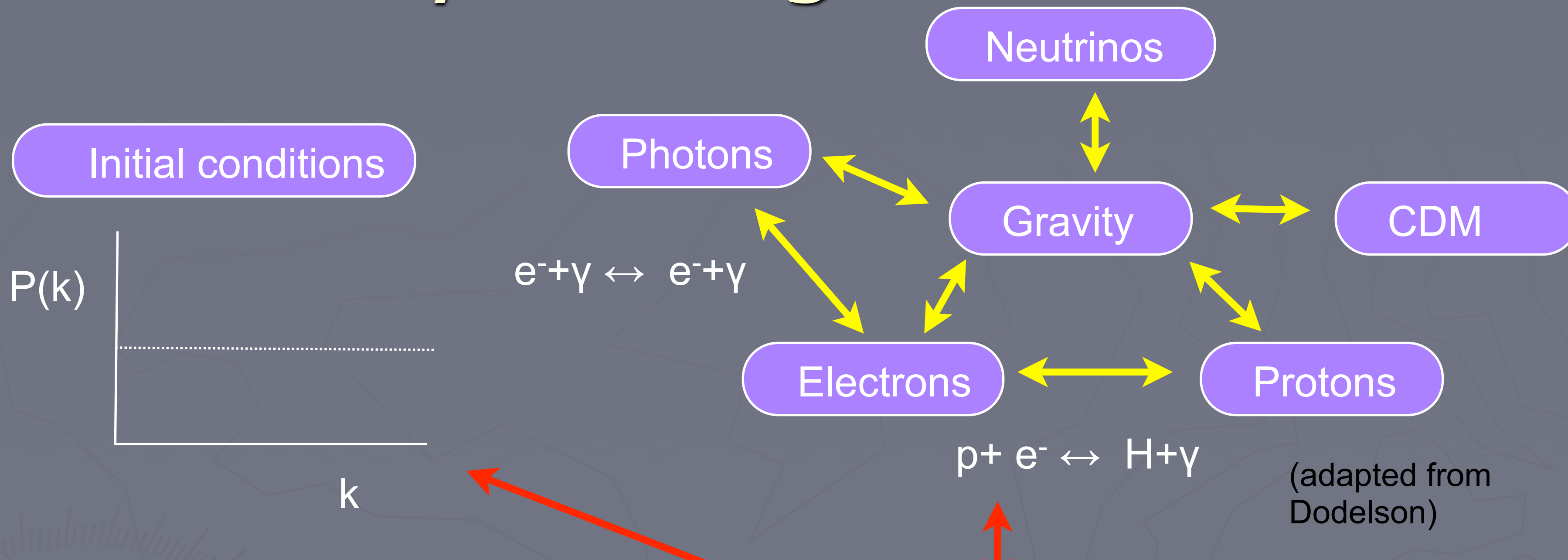
Features of power spectrum

- Sources of anisotropy
  - Sachs Wolfe - Potential fluctuations at LSS (photon redshifting)
  - Density - $\rho \propto T^4$
  - Velocity - Doppler shift
  - ISW - Changing gravity along line of sight
- Secondaries
  - Lensing, SZ
- Physical features
  - Damping envelope
  - Equally spaced acoustic peaks



(Lineweaver, 1997)

Physical ingredients



$$C_l = 4\pi \int d\ln k\, \mathcal{P}_s(k) |\Delta_l(k,\tau_0)|^2$$

- Gravity = Linearized Einstein equations (solve in Fourier space as modes decouple)
- Continuity/Euler equations for fluid compts, Boltzmann for photons/neutrinos
- Large set of coupled differential equations!
- Keep in mind this picture when dissecting a CMB code!

Basic theory

- Metric perturbations (NB gauge choice):

$$ds^2 = -dt^2 + a(t)^2(\delta_{ij} + h_{ij}dx^i dx^j)$$

- Fluid perturbations :

$$\begin{aligned} T^0{}_0 &= -(\rho + \delta\rho) \\ T^0{}_i &= (\rho + P)v^i \\ T^i{}_j &= (P + \delta P)\delta^i{}_j + \Pi^i{}_j \end{aligned}$$

- CMB anisotropies (assume sharp recombination):

$$\frac{\Delta T}{T} \sim \int \dot{h}_{ij} n^i n^j d\tau + \frac{1}{4}\delta_\gamma(t_\star) + n \cdot v_\gamma(t_\star)$$

POTENTIAL **DENSITY** **VELOCITY**

$$\frac{\Delta T}{T} = \frac{\delta\Phi}{3}$$

(Good reference = Ma and Bertschinger)

Oscillations

$$c_\mathrm{s} = \frac{c}{\sqrt{3}}$$

- Tight coupling, no gravity, ignore baryons

$$\ddot{\delta}_\gamma + c_\mathrm{s}^2 k^2 \delta_\gamma = 0$$

$$\delta_\gamma = \delta_\gamma(0)\cos(c_\mathrm{s} k \tau)$$

$$k_\mathrm{n} = \frac{(n+1)\pi}{c_\mathrm{s}\tau_\star}$$

- Add velocity perturbations (directional effect along k vector)

$$v_\gamma = -\frac{3}{4k}\dot{\delta}_\gamma$$

- Add baryons
  - Drag (Changes relative peak heights, also lowers sound speed)

$$c_s = \frac{c}{\sqrt{3(1+R)}}$$

$$R = \frac{3\rho_\mathrm{b}}{4\rho_\gamma}$$

  - Damping envelope (τ_D time-scale associated with mean free path)

$$\ddot{\delta}_\gamma + 2k^2\tau_\mathrm{D}\dot{\delta}_\gamma + c_\mathrm{s}^2 k^2 \delta_\gamma = 0$$

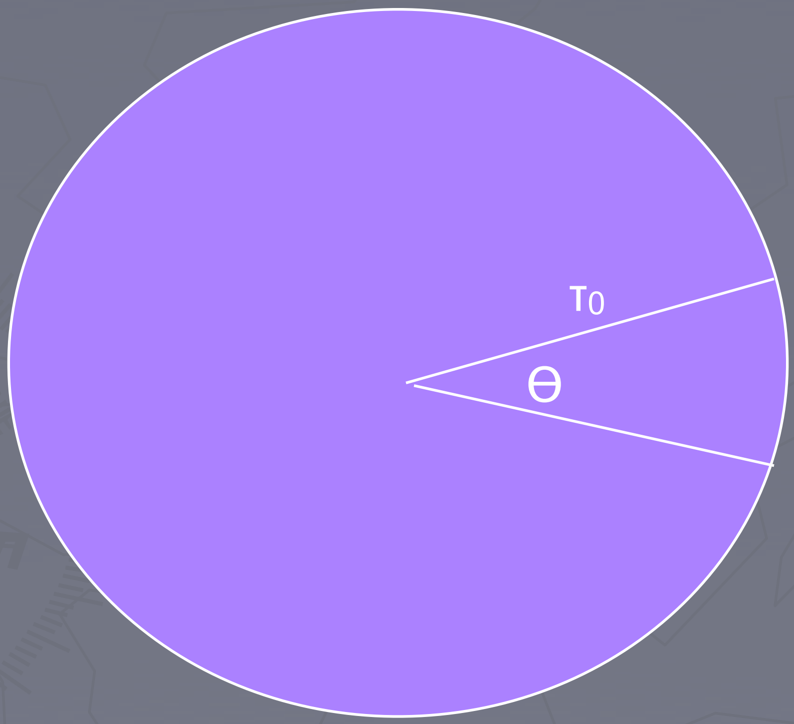
$$\delta_\gamma = e^{-k^2\tau_\star\tau_\mathrm{D}}\delta_\gamma^\mathrm{TC}$$

Projection

- Project spatial anisotropies in k space to angular anisotropies
- τ_0 sensitive to expansion since recombination (dark energy, spatial curvature)

$$\ell \approx k\tau_0$$

$$\ell_{\rm peak} = \frac{\pi\tau_0}{c_{\rm s}\tau_\star} \approx 225$$

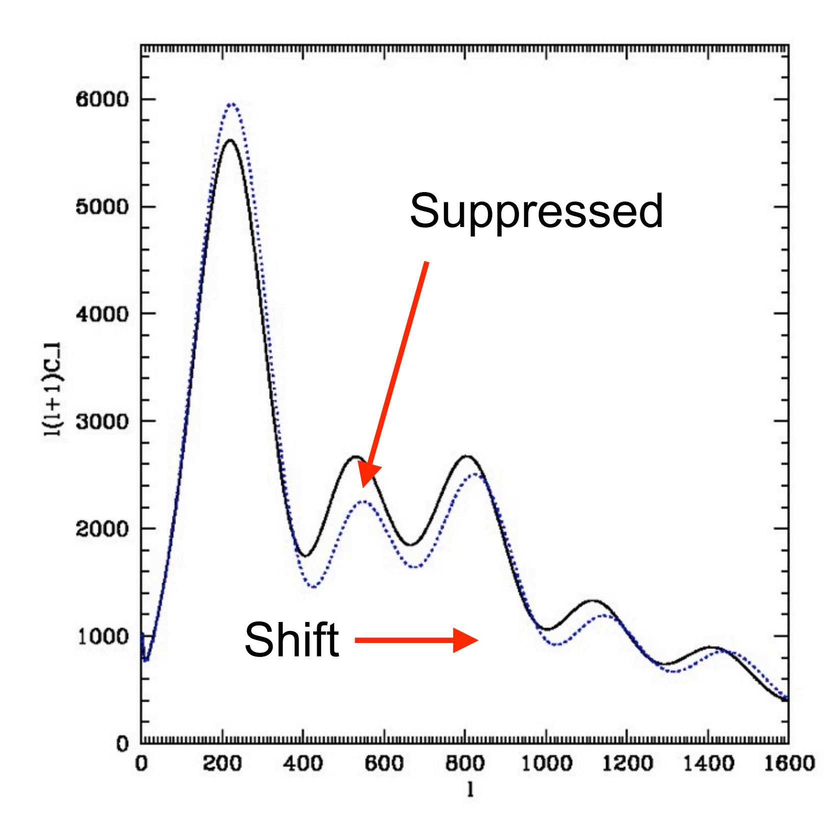


Time to test the theory!

- Go to
  - http://lambda.gsfc.nasa.gov/toolbox/tb_camb_form.cfm
  - Or search google for "CAMB online interface"
- Generate basic temperature power spectrum and save in separate window
- Remember - we know parameters to better than 10 %

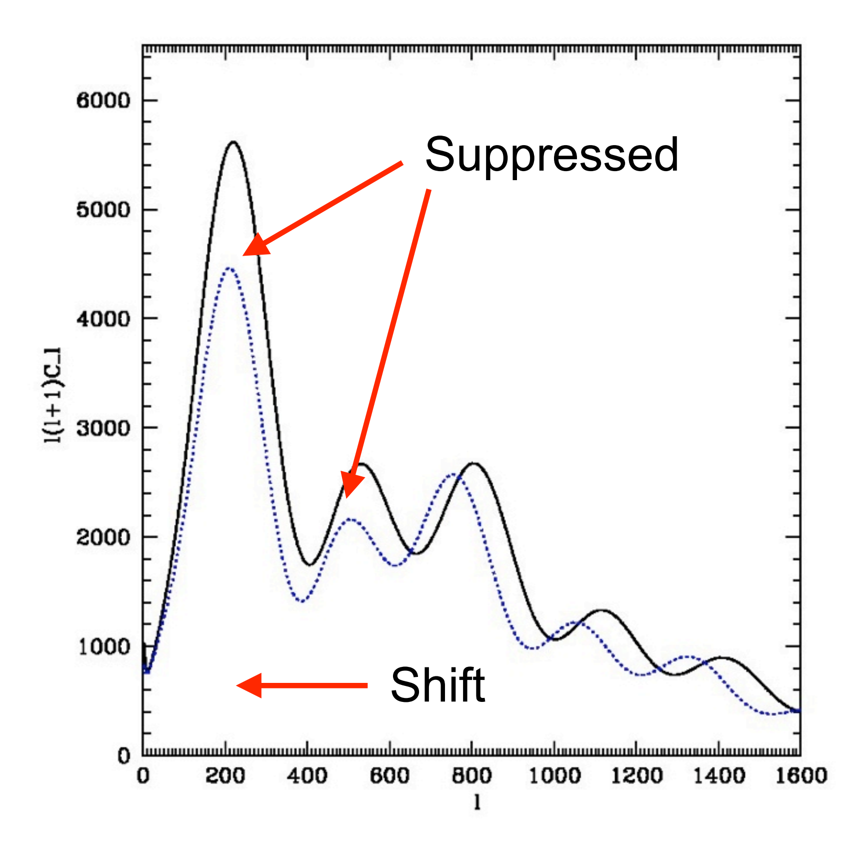
Baryons

- Increase baryon fraction - I used $\Omega_B h^2 = 0.03$
  - 2nd peak suppressed relative to first/third due to baryon loading
  - Peaks moved to larger l as sound speed reduced



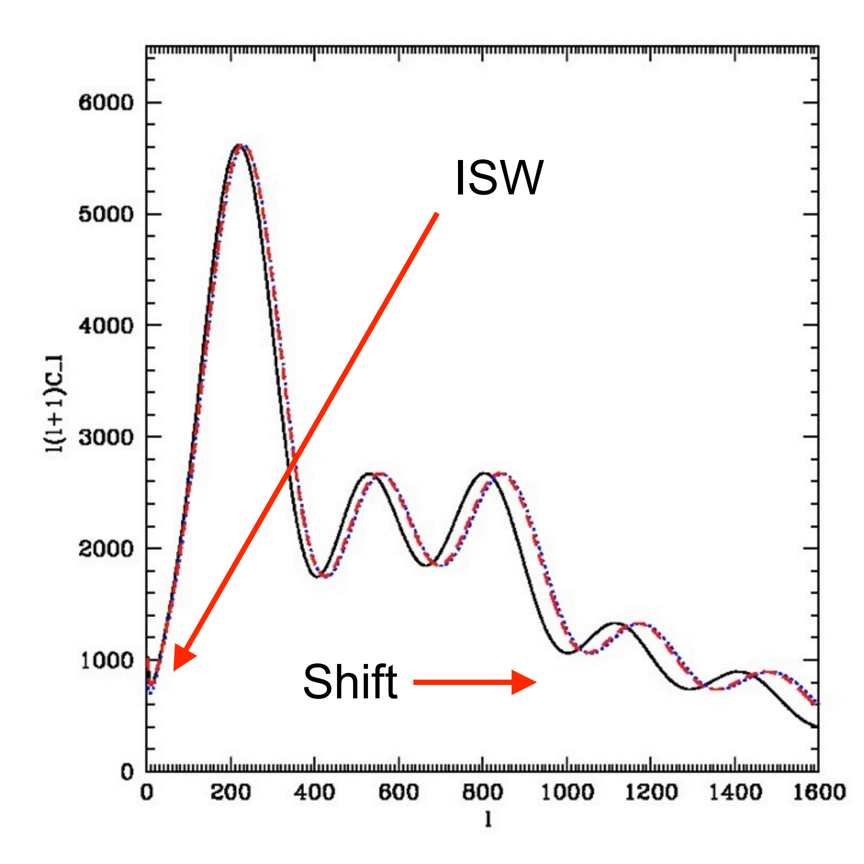
Cold dark matter

- Increase CDM fraction - I used $\Omega_{CDM} h^2 = 0.2$
  - Higher CDM - earlier matter/radiation equality
  - Modes have higher amplitude when they enter horizon in radiation era (large scales suppressed)
  - Higher matter shifts peaks to larger scales



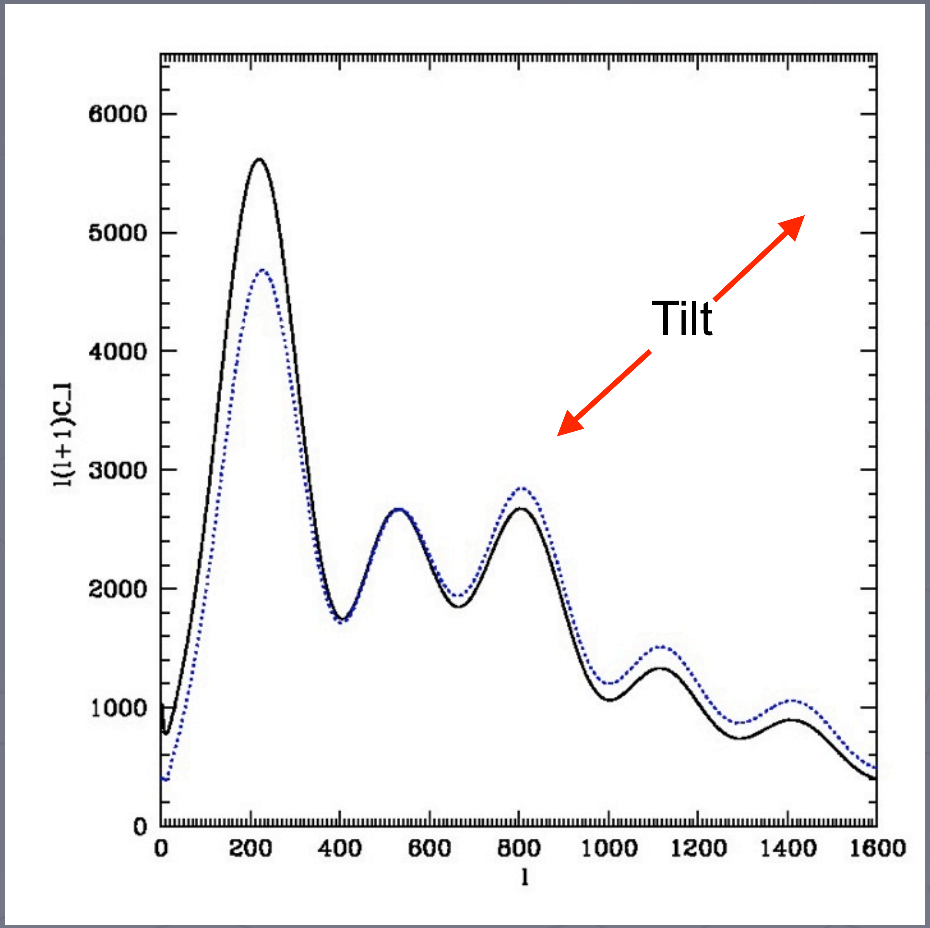
Projection effects

- Lower w - I used w = -2, Increase curvature I used $\Omega_k = 0.03$ (open)
  - Open universe lines diverge
  - Both increase angular diameter distance



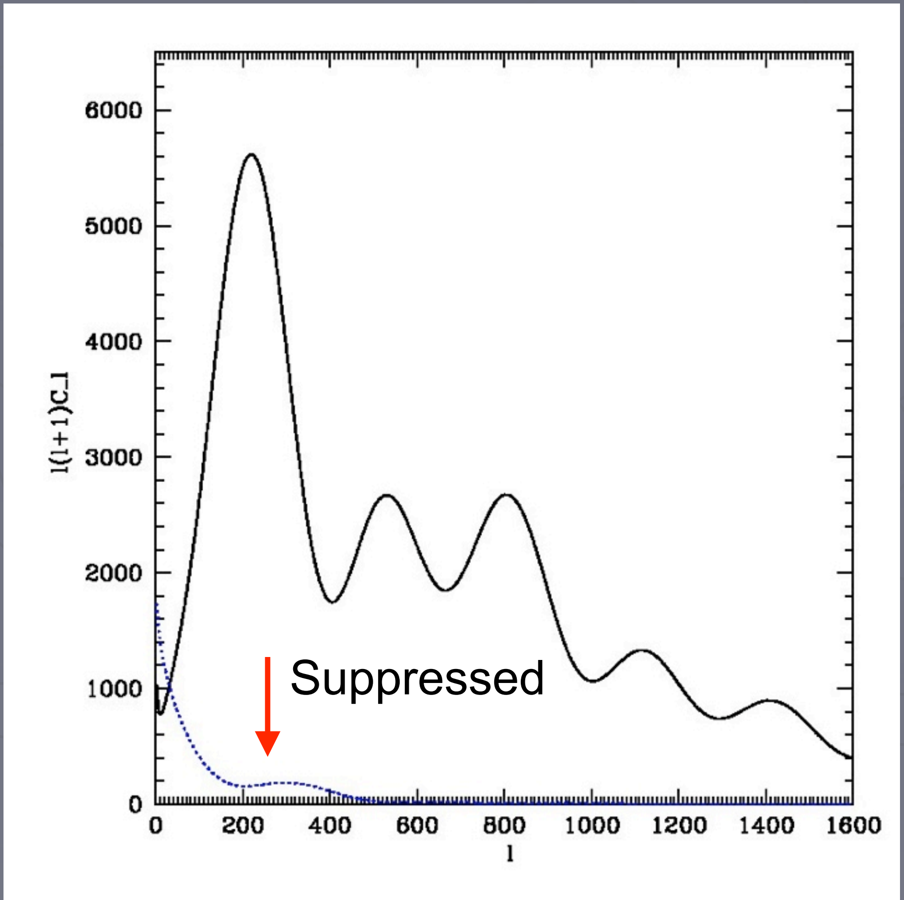
Initial spectrum

- Increase spectral index - I used $n_s = 1.2$ (NB 1 is scale invariant)
  - Power increased on small relative to large scales (blue tilt)



Initial fluctuations

- Change initial fluctuation type - I used CDM isocurvature mode
  - Small scale power suppressed



What do we learn?

- Primary anisotropies
  - Cosmological parameters
  - How the universe recombined
  - Fundamental physics - dark energy, neutrinos
  - Origin of initial perturbations - gaussian, gravitational, power spectrum
  - Extensions to standard model - geometry and topology
- Secondary anisotropies
  - ISW
  - Reionization
  - Clusters - SZ and lensing

Dissecting CMB code

- Trade off between accuracy/time
- Line of sight integration method

$$C_l = 4\pi \int d\ln k \mathcal{P}_s(k) |\Delta_l(k,\tau_0)|^2$$

$$\Delta_\ell(k,\tau_0) = \int_0^{\tau_0} S(k,\tau) j_\ell(k(\tau_0-\tau)) d\tau$$

Physics

- Time saving
  - Less k integrations
  - Sources tightly coupled - no need to solve for high l

Initialization

Source k values

↓

Integration k values

↓

Bessel functions

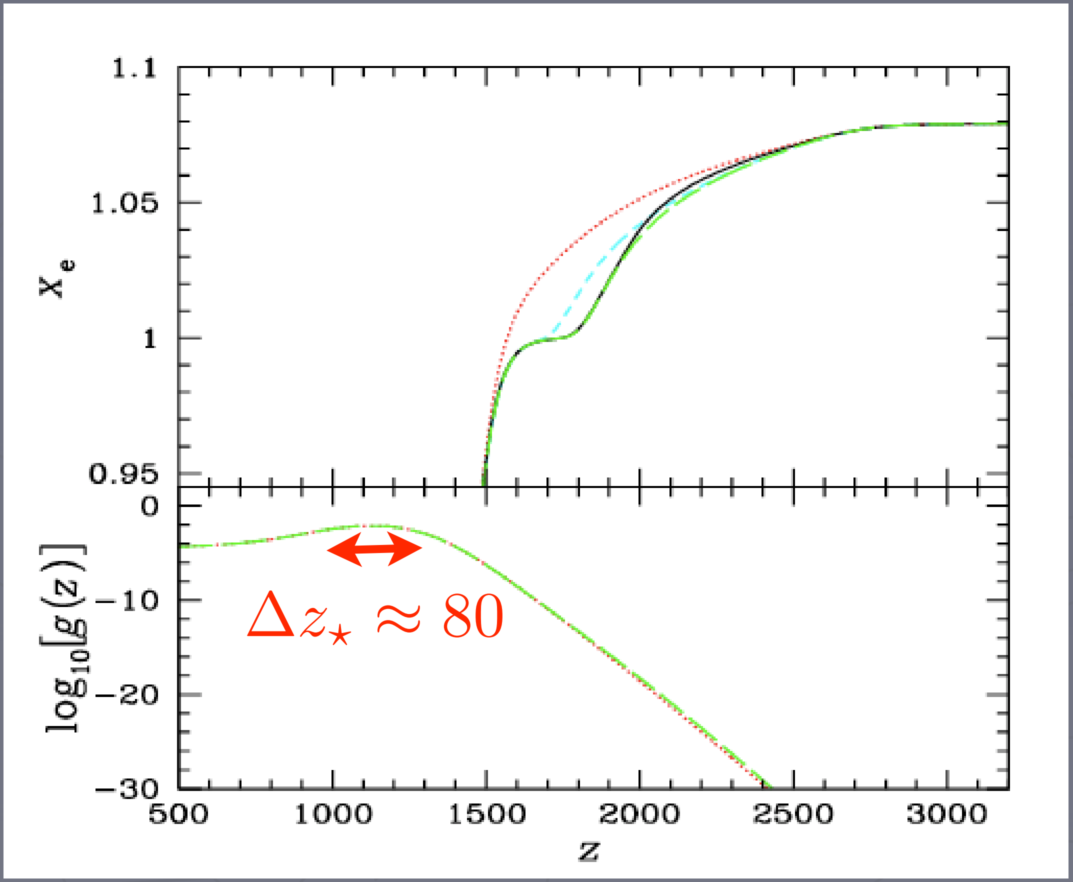
↓

Recombination history

Recombination

- Differential visibility:

$$g(z) = \dot{\kappa} e^{-\kappa}$$



(Wong, Moss and Scott, 2008)

Dissecting CMB code

Initial conditions

Loop over k

Evolution

Propagate equations

Evaluate sources

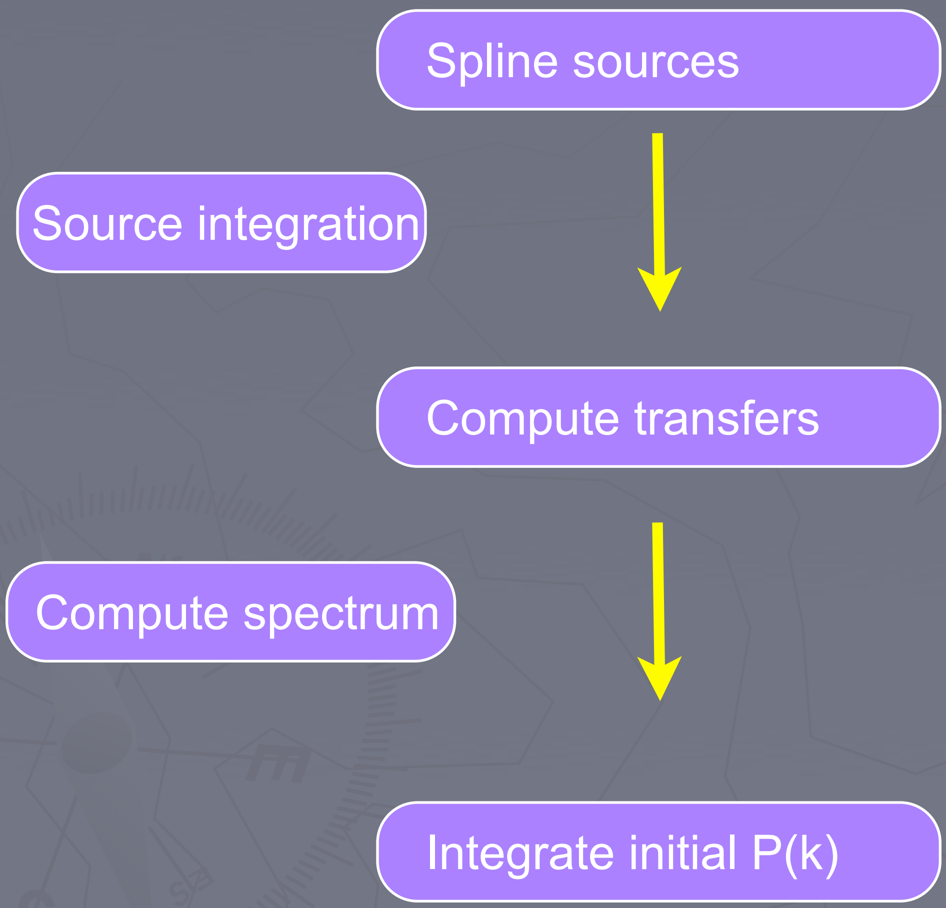
- Save time by evolving from a = 10^{-8}
- Example of gravitational curvature perturbation:

$$\delta_\gamma = \frac{1}{4}(k\tau)^2$$

$$\delta_{\rm CDM} = \delta_{\rm B} = \frac{1}{3}(k\tau)^2$$

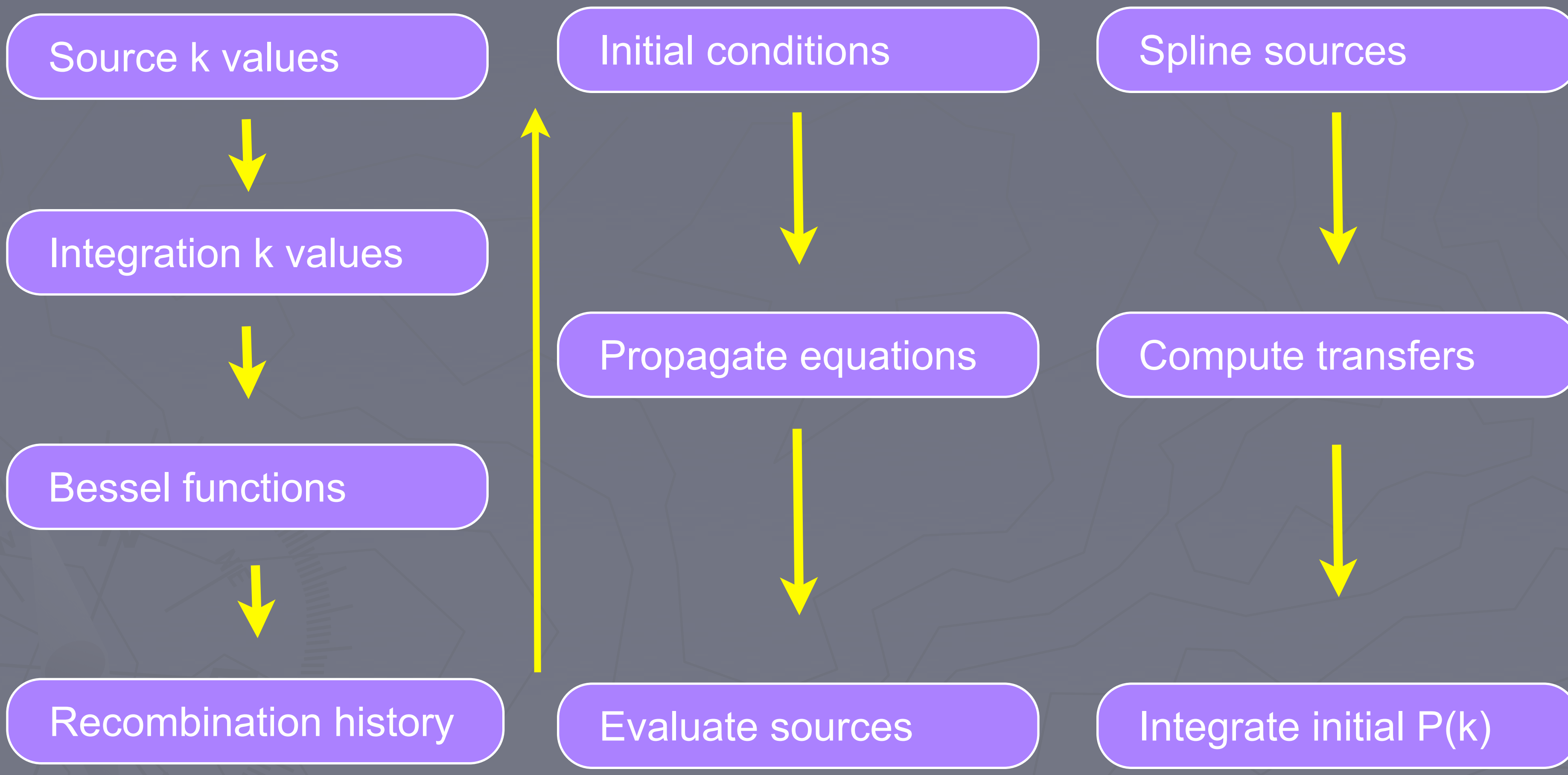
- Propagate coupled differential equations - codes usually compute scalar, vector, tensor modes separately
- Evaluate sources for line of sight method

Dissecting CMB code



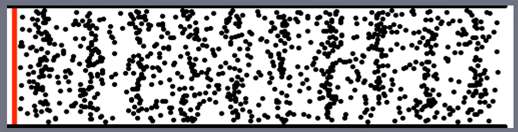
- Spline sources for bessel function integration
- Integrate over time with bessel functions
- Integrate over k modes with inititial power spectrum

Put it together

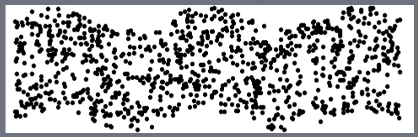


Dark energy example

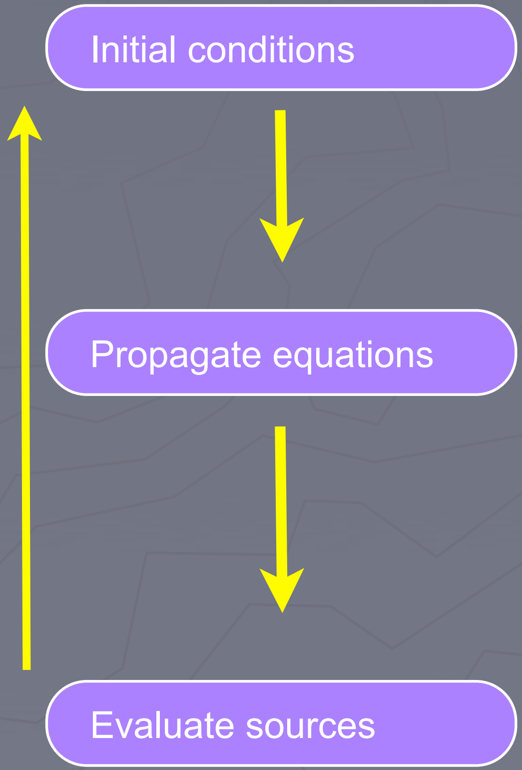
- Modified fluid equations due to shear modulus μ



$$c_{\rm s}^2 = w + \frac{4}{3}c_{\rm v}^2 \qquad c_{\rm v}^2 = \frac{\mu}{\rho + p}$$



- Changes:
  - Initial conditions for fluid
  - Change evolution equations
  - Alter line of sight integral
- Total lines to change - around 10
- NB alot more work goes into it than that!



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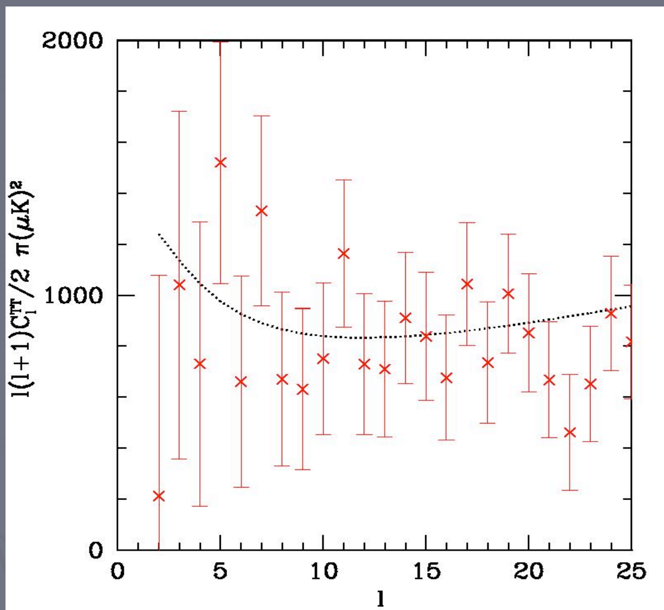

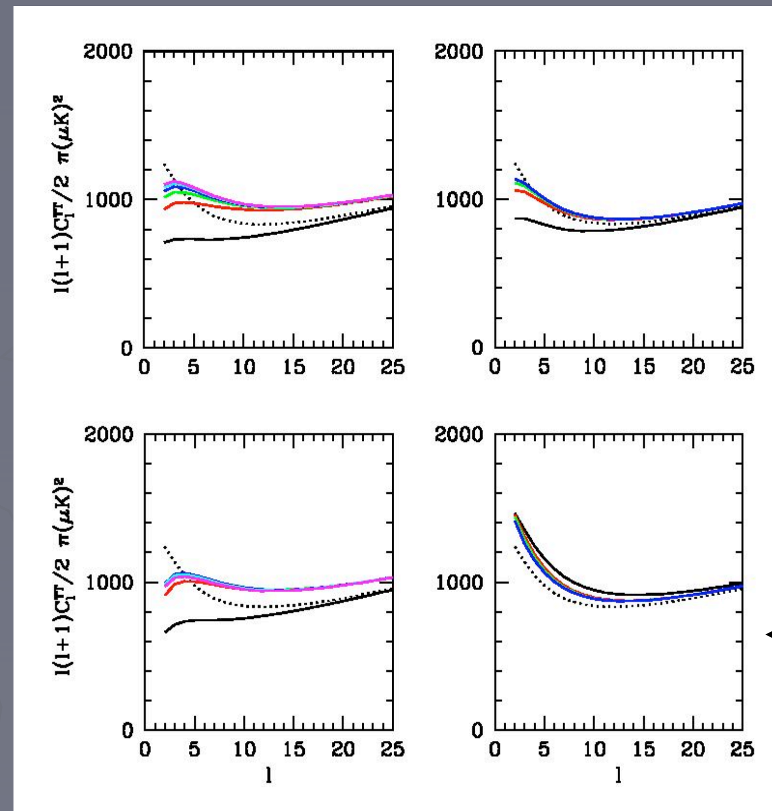

Scalar field (non-zero internal entropy)

Elastic (non-zero anisotropic stress)

w = -1/3

w = -2/3

(Battye and Moss, 2007)

Conclusions/Discussion

- Conclusions
  - The CMB has allowed us to enter an era of precision cosmology - need precise computational tools
  - Important to still have intuition in using a black box tool
  - Although codes are lengthy, the structure is easy to understand
  - Many extra physical processes can be tested in this way
- Discussion
  - Do people here use CMB codes to model extra physics?
  - What is required of future CMB codes e.g Planck and beyond?