Digital Active Feedback

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Abstract

TES bolometers operated under strong electrothermal feedback require a constant voltage bias. This requires a low input impedance amplifier, such as a SQUID. However, by providing digital active feedback, I show that it is also possible to provide a constant voltage bias to the bolometer while relaxing the requirement of a low input impedance amplifier.

Contents

1 Introduction

SQUIDs have been the amplifiers of choice for the readout of TES bolometers. The reason is that a SQUID amplifier has a low input impedance when compared to the operating resistance of the bolometer. Removing this requirement could allow for new types of amplifiers to be used.

TES bolometers respond to incident power with a time constant $\tau_{bolo} = \frac{C}{G}$ where C is the heat capacity of the TES and G is the thermal conductance to the heat sink. Actively controlling the voltage across the bolometer on time scales shorter than τ_{bolo} (digital active feedback) is a mechanism which can voltage bias a TES bolometer while allowing for an amplifier input impedance in series with the bolometer.

In this document, I will present a Simulink simulation of a TES bolometer functioning with and without digital active feedback.

2 TES Simulation

Simulating a TES bolometer requires a model, such as the one presented in Section 3.3.1 of Trevor's thesis. Schematically, the power balance for a bolometer can be written as

$$\left(P_{sky} + \frac{V_{bias}^2}{R}\right)\frac{1}{1+i\omega\tau} = G\Delta T \tag{1}$$

where R is the bolometer resistance, P_{sky} is the incident optical power, $\frac{V_{bias}^2}{R}$ is the absorbed electrical power, G is the some thermal conductance to the heat sink, $\tau = \frac{C}{G}$ is the bolometer time constant and ΔT is the temperature difference to the heat sink.

I use an ad hoc model of the TES transition

$$\frac{R}{1\Omega} = \frac{1}{2} + \frac{1}{\pi}\arctan\left(\frac{T - T_c}{T_0}\right) \tag{2}$$

which produces the transition shown in Figure 1.

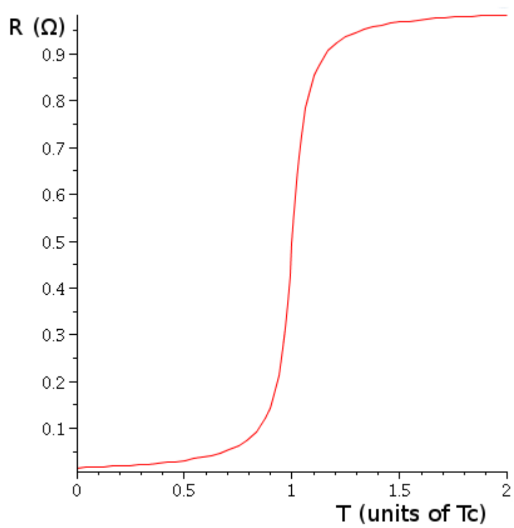

Figure 1: An approximation to the TES transition to superconductivity.

Combining Equations 1 and 2, I have simulated a bolometer, as shown in Figure 2.

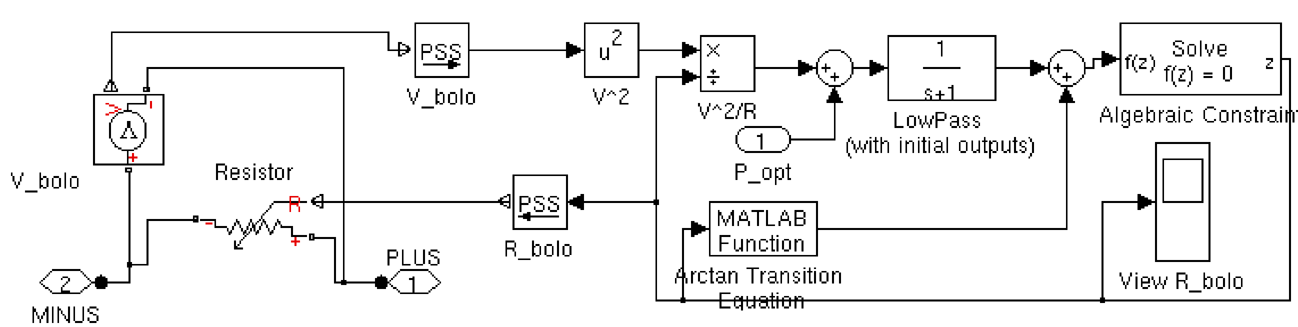

Figure 2: Simulation of a TES bolometer in Simulink.

I integrated this simulated bolometer into a setting shown in Figure 3.

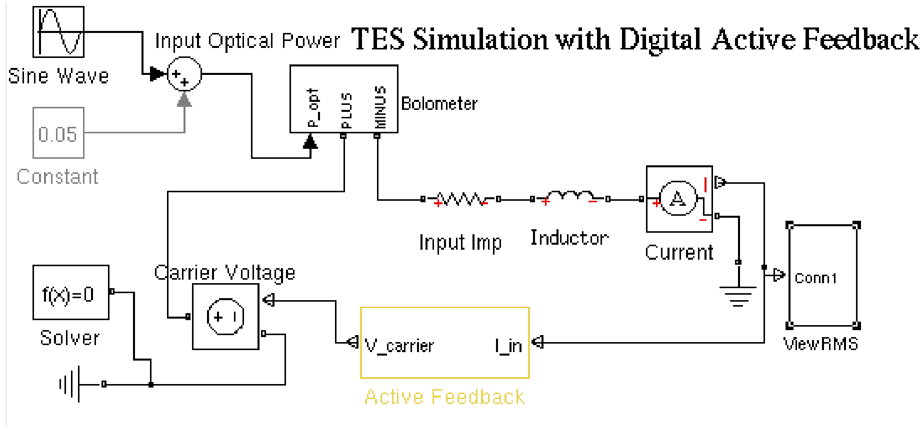

Figure 3: Simulation of a bolometer operating under sinusoidally varying optical power, in series with a readout related complex impedance.

In order to verify the accuracy to which the simulated TES reproduces known characteristic features of voltage biased TES bolometers, I then proceded to varying the voltage across the bolometer and measuring the resulting current. Such an "I-V" curve is shown in Figure 4, accurately reproducing the qualitative features of a TES bolometer.

At low voltage bias, the bolometer is superconducting, leaving only the input impedance of the amplifier as a resistive element. The characteristic $IV = constant$ shows the portion of the IV curve in which the power delivered to the bolometer is a constant: the power required to keep the bolometer at temperature T_c. Deviations from the $IV = constant$ relation indicate instabilities in the bolometer. At high applied voltages the bolometer has a resistance of 1Ω, acting as a purely resistive element.

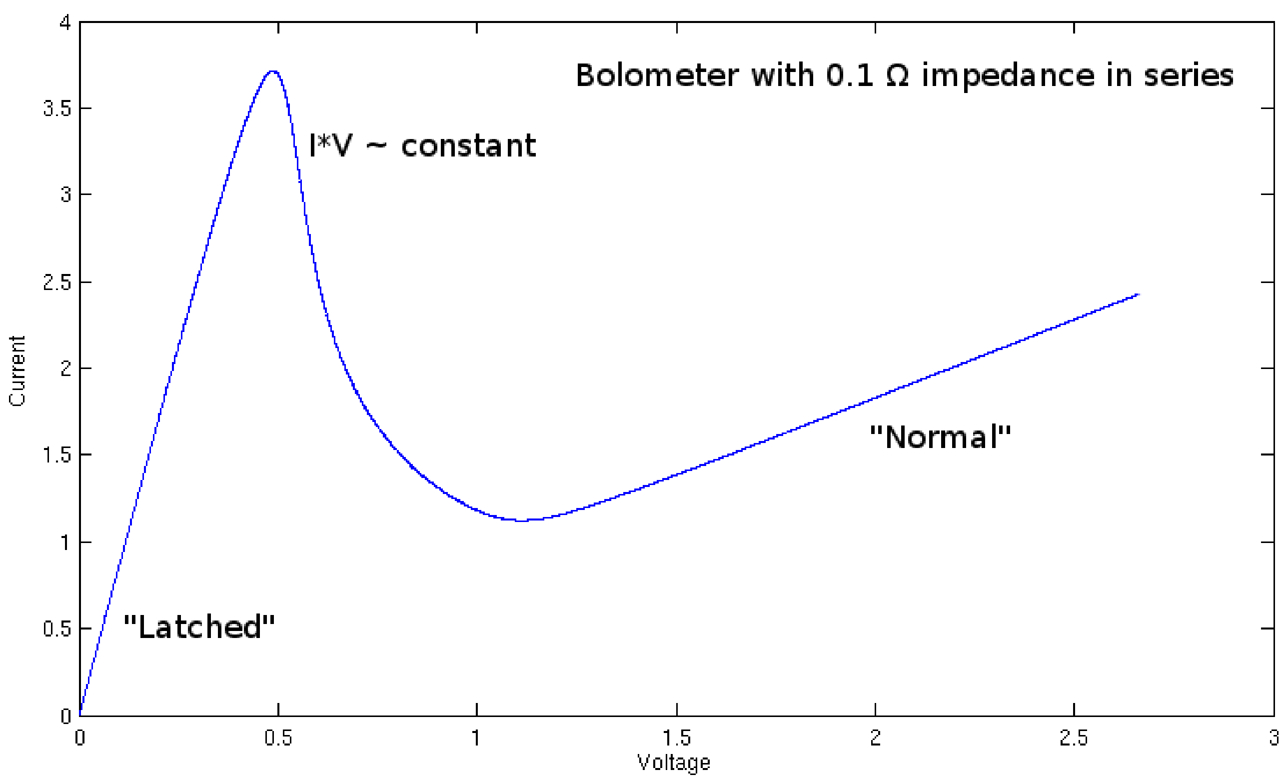

Figure 4: I-V curve for the simulated TES bolometer with a 0.1Ω resistor in series.

3 Active Feedback Simulation

Figure 3 shows a system which takes in the measured current through the bolometer and generates the relevant carrier voltage. Without active feedback, the carrier voltage is generated as shown in Figure 5.

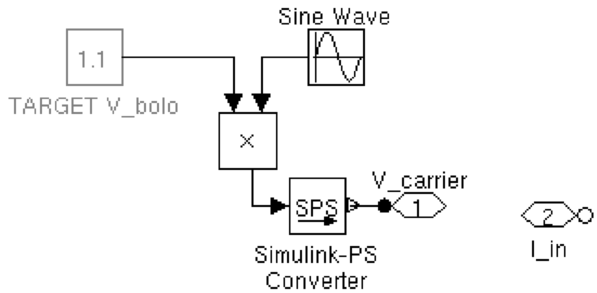

Figure 5: Carrier generation without active feedback.

However, given that the readout system has complex impedance $z_s = R_s + iX_s$ we can use our knowledge of the circuit to write that

$$V_{bolo} = \left[\sqrt{\left(\frac{V_{carrier}}{I}\right)^2 - X_s^2} - R_s\right] I \tag{3}$$

Note that to measure R_s and X_s as a function of frequency, one could latch the bolometer into superconductivity, leaving exactly z_s as the complex impedance of the system.

We can now provide a PID control system to require V_{bolo} to remain constant. When adding this digital active feedback, the simulation is shown in Figure 6.

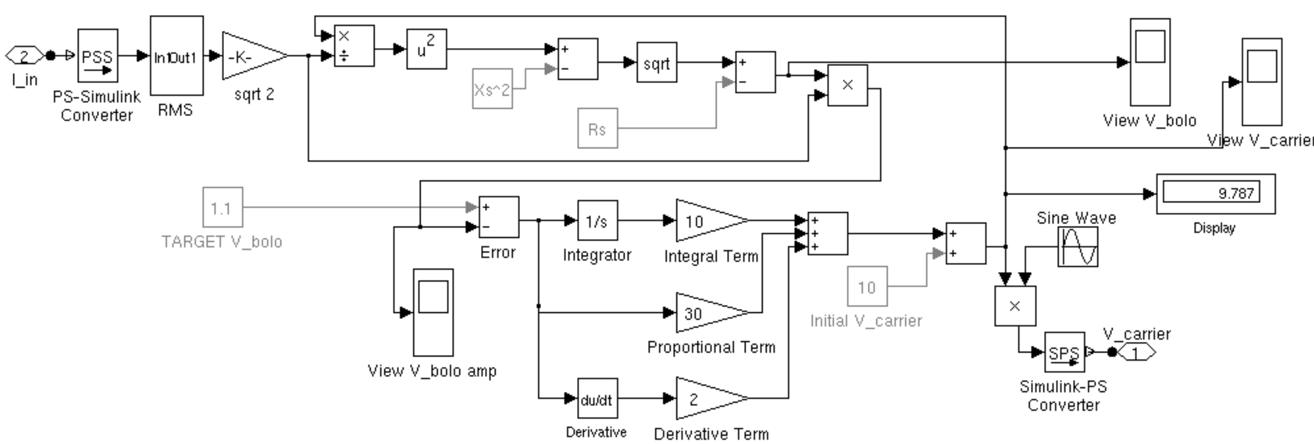

Figure 6: Carrier generation using active feedback in the form of a PID controller.

The three gain parameters of the PID controller can then be chosen to optimize the performance of the active feedback mechanism. Note that we require the condition $\tau_{PID} \ll \tau_{TES}$, in other words the control must occur at shorter timescales than the TES can respond. The amount of PID parameter optimization required depends on how well that condition is met. In my simulation, in order to run in a reasonable amount of time (typically 6 minutes), I satisfied the $\tau_{PID} \ll \tau_{TES}$ condition to a lesser extent than realistically possible in a physical system.

4 Results

I will now demonstrate the performance of active feedback within this simulated system with and without active feedback for two values of amplifier input impedance.

4.1 Without Active Feedback

Figures 7 and 8 show the bolometer response to sinusoidally varying optical power. The bolometer changes its resistance sinusoidally in response to the large changes in applied optical power. Clearly a large input impedance violates the stability condition of a constant voltage bias and the bolometer latches.

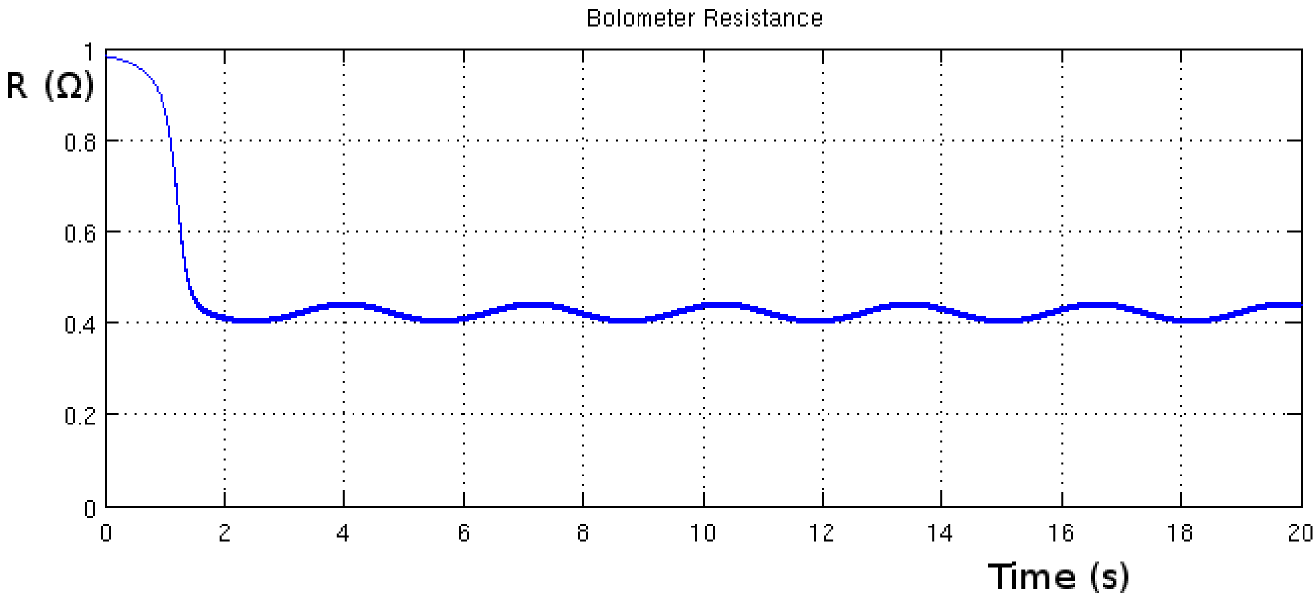

Figure 7: Bolometer resistance while responding to sinusoidal optical power without active feedback with a 0.1Ω amplifier input impedance.

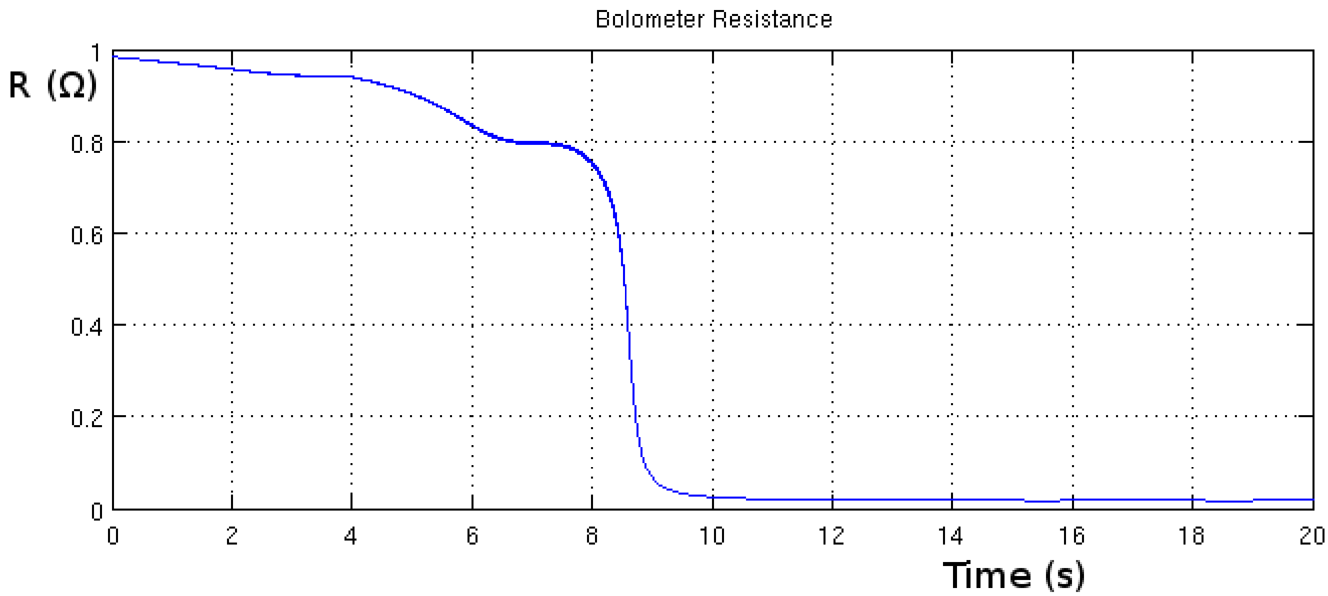

Figure 8: Bolometer resistance while responding to sinusoidal optical power without active feedback with a 2Ω amplifier input impedance. Note that the bolometer is unstable in the sense that it is easily driven normal or superconducting by changed in V_{bias} or optical power.

4.2 With Active Feedback

Figures 9 and 10 show the bolometer response to sinusoidally varying optical power with the digital active feedback mechanism in place. Note that despite the increase in amplifier input impedance, as long as the timescale of the active feedback mechanism is much shorter than that of the bolometer, the stability condition of constant voltage bias is still met and the bolometer functions correctly.

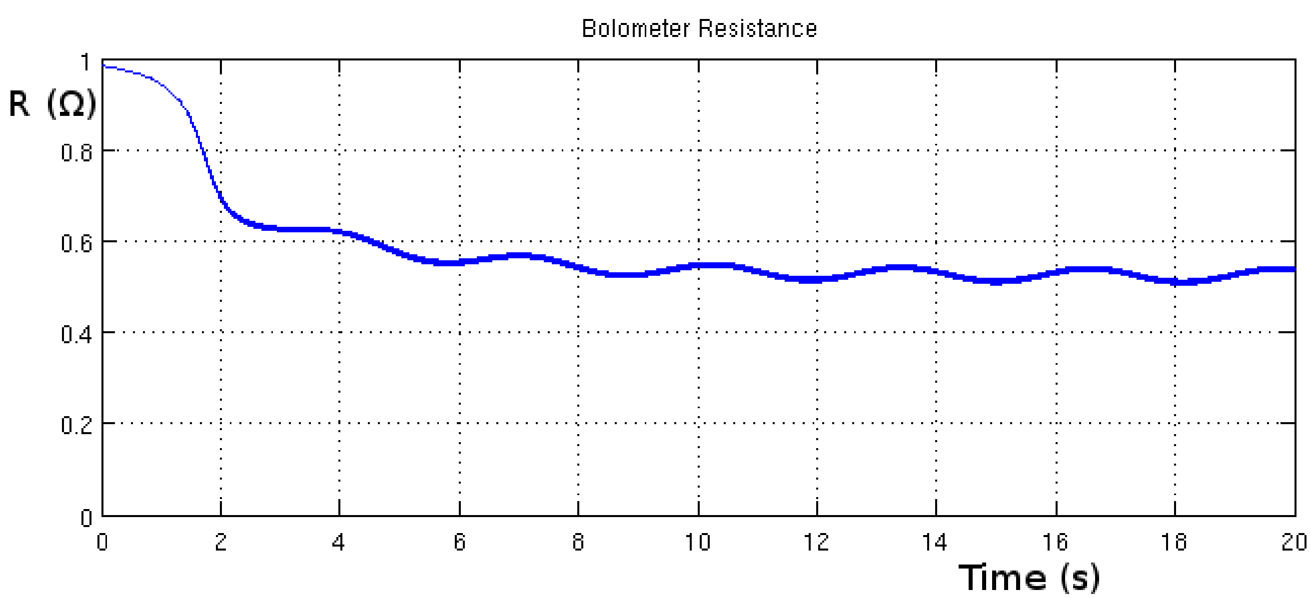

Figure 9: Bolometer resistance while responding to sinusoidal optical power with active feedback with a 0.1Ω amplifier input impedance.

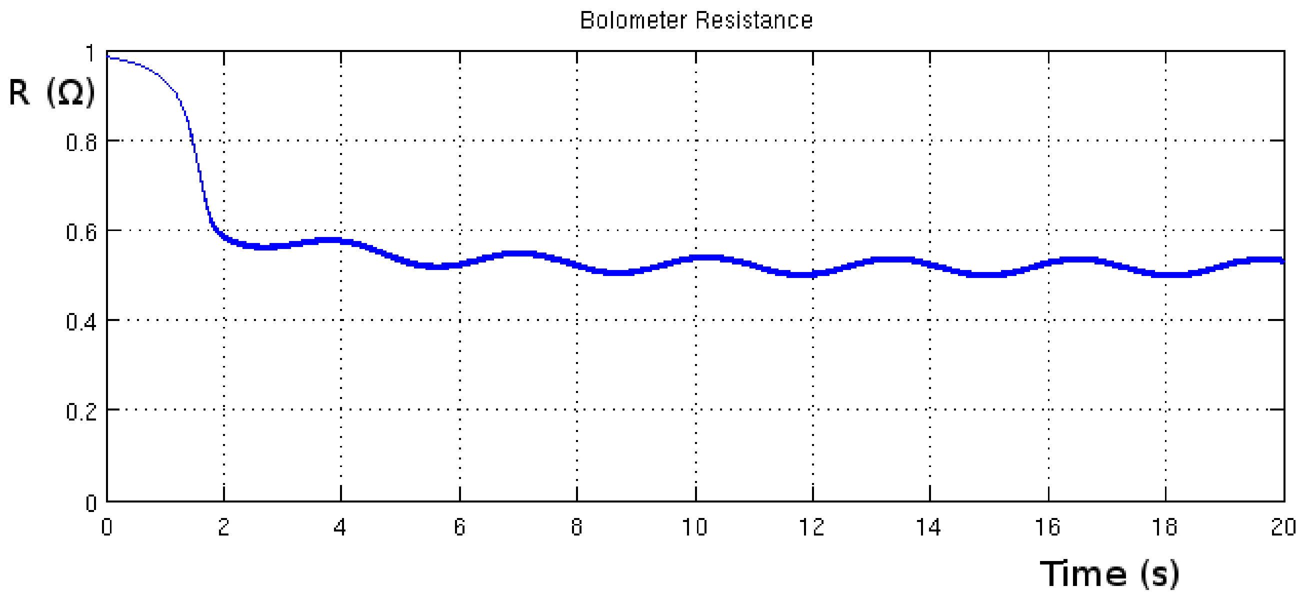

Figure 10: Bolometer resistance while responding to sinusoidal optical power with active feedback with a 2Ω amplifier input impedance.

5 Data Products

Bolometers are used to measure optical power. In existing systems, the current measured by the amplifier is often downsampled to ~ 100 Hz and stored to disk. The resulting data product is then proportional to the measured sky power, the relevant proportionality constant being the bias voltage. However, in a system with digital active feedback, the measured current is no longer proportional to the incident optical power. The relevant quantity to be stored to disk is the product $I \cdot V_bolo$.

Simulations show that a significant misestimation of any one of the parameters I, X_s or R_s results in an erroneous measurement of V_bolo. Therefore, storing I and V_bolo to disk separately allows for more information to be recovered about the active feedback circuit.

6 Noise

Any noise manifesting as "resistance noise", such as shot noise from incident photons or phonon noise on the bolometer, are rolled off by the TES time constant. Therefore, the noise properties are unchanged under active feedback.

In contrast, Johnson noise on the TES can be interpreted as voltage noise. Therefore only noise near the carrier frequency (to which the demodulator is sensitive) is relevant.

Section in progress...

7 Stability

The previous sections assumed that the RMS current in the system could be measured. However, a lock-in demodulator typically locks into the phase of the current and measures only that component. If we define the carrier voltage to be

$$V_{carrier} = |V|e^{i\omega t} \tag{4}$$

then we can define the current in the system to be

$$I = |I|e^{i\omega t+\phi} \tag{5}$$

where

$$\phi = \arctan\left[\frac{X_s}{R_s + R_{bolo}}\right] \tag{6}$$

This phase shift can be "locked onto" by the demodulator. However, if we perturb the phase $\phi \to \phi+\delta\phi$ then our measured current will be affected. Hence, the estimate of V_{bolo} in the control loop will be erroneous.

Under the perturbation $\phi \to \phi + \delta\phi$, the effective result of the control loop is $R_{bolo} \to R_{bolo} + \delta R_{bolo}$. Logically following the action of the control system shows that $\delta R_{bolo} < 0$, regardless of the sign of $\delta\phi$.

The perturbation in the current is

$$\delta I = \frac{V_{bolo}}{R_{bolo} + R_s + iX_s} - \frac{V_{bolo}}{R_{bolo} + \delta R_{bolo} + R_s + iX_s} \tag{7}$$

combining terms yields

$$\delta I = \frac{V_{bolo}\delta R_{bolo}\left(\left[X_s^2 - R^2 - \delta R_{bolo}\right] + i\left[X_s\left(2R + \delta R_{bolo}\right)\right]\right)}{\left((R + \delta R_{bolo})^2 + X_s^2\right)\left(R^2 + X_s^2\right)} \tag{8}$$

However, we are only sensitive to one component. If we let

$$\mathbf{u}(z) = \begin{bmatrix} \mathbb{R}(z) \\ \mathbb{I}(z) \end{bmatrix} \tag{9}$$

where z is some complex number then we can project out the relevant component by writing

$$\mathbf{u}(I_{||}) = \mathbf{u}\left(\frac{I}{|I|}\right)\mathbf{u}^{\mathrm{T}}\left(\frac{I}{|I|}\right)\mathbf{u}(\delta I) \tag{10}$$

which, after some algebra, becomes (along the direction of I)

$$\delta I = \frac{V_{bolo}\delta R_{bolo}\left(R_s + R_{bolo}\right)\left(\left(R_s + R_{bolo}\right)^2 - X_s^2\right)}{\left(\left(R_s + R_{bolo}\right)^2 + X_s^2\right)^2} \tag{11}$$

where I have ignored terms of order δR_{bolo}^2. Note that the effect of an instantaneous phase perturbation $\delta\phi$ is an instantaneous decrease in the measured I. If the active feedback then provides $\delta I > 0$, the system *must* be stable.

Using Equation 11, this shows that $R_s + R_{bolo} > X_s$ guarantees stability. In other words when the phase of the current is within 45° of the phase of the carrier voltage, stability is guaranteed. Simulations show that even when $X_s > R_s + R_{bolo}$, stability can still be achieved under certain PID parameters and/or optical signals.

8 Conclusion

Provided it occurs on fast timescales, the active feedback mechanism allows for a constant voltage bias to the bolometer, despite violation of the $z_{amp} \ll z_{bolo}$ condition.