Digital Active Feedback

Tijmen de Haan

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Abstract

TES bolometers operated under strong electrothermal feedback require a constant voltage bias. This requires a low input impedance amplifier, such as a SQUID. However, by providing digital active feedback, I show that it is also possible to provide a constant voltage bias to the bolometer while relaxing the requirement of a low input impedance amplifier.

Contents

1 Introduction

SQUIDs have been the amplifiers of choice for the readout of TES bolometers. The reason is that a SQUID amplifier has a low input impedance when compared to the operating resistance of the bolometer. Removing this requirement could allow for new types of amplifiers to be used.

TES bolometers respond to incident power with a time constant $\tau_{bolo} = \frac{C}{G}$ where C is the heat capacity of the TES and G is the thermal conductance to the heat sink. Actively controlling the voltage across the bolometer on time scales shorter than τ_{bolo} (digital active feedback) is a mechanism which can voltage bias a TES bolometer while allowing for an amplifier input impedance in series with the bolometer.

In this document, I will present a Simulink simulation of a TES bolometer functioning with and without digital active feedback.

2 TES Simulation

Simulating a TES bolometer requires a model, such as the one presented in Section 3.3.1 of Trevor's thesis. Schematically, the power balance for a bolometer can be written as

$$\left(P_{sky} + \frac{V_{bias}^2}{R}\right)\frac{1}{1+i\omega\tau} = G\Delta T \tag{1}$$

where R is the bolometer resistance, P_{sky} is the incident optical power, $\frac{V_{bias}^2}{R}$ is the absorbed electrical power, G is the some thermal conductance to the heat sink, $\tau = \frac{C}{G}$ is the bolometer time constant and ΔT is the temperature difference to the heat sink.

I use an ad hoc model of the TES transition

$$\frac{R}{1\Omega} = \frac{1}{2} + \frac{1}{\pi}\arctan\left(\frac{T-T_c}{T_0}\right) \tag{2}$$

which produces the transition shown in Figure 1.

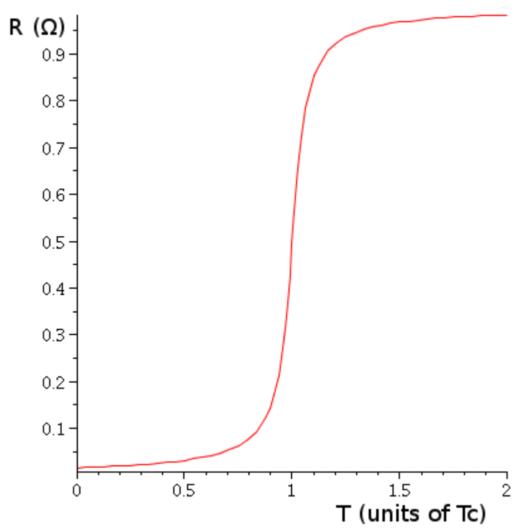

Figure 1: An approximation to the TES transition to superconductivity.

Combining Equations 1 and 2, I have simulated a bolometer, as shown in Figure 2.

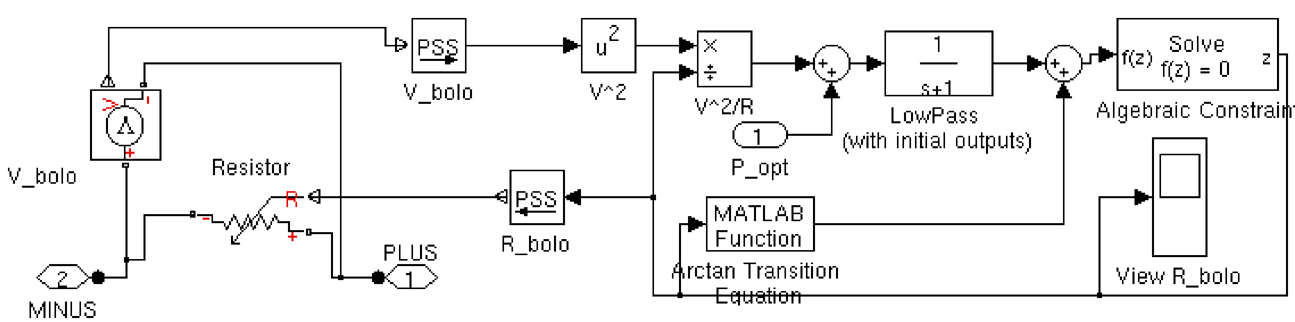

Figure 2: Simulation of a TES bolometer in Simulink.

I integrated this simulated bolometer into a setting shown in Figure 3.

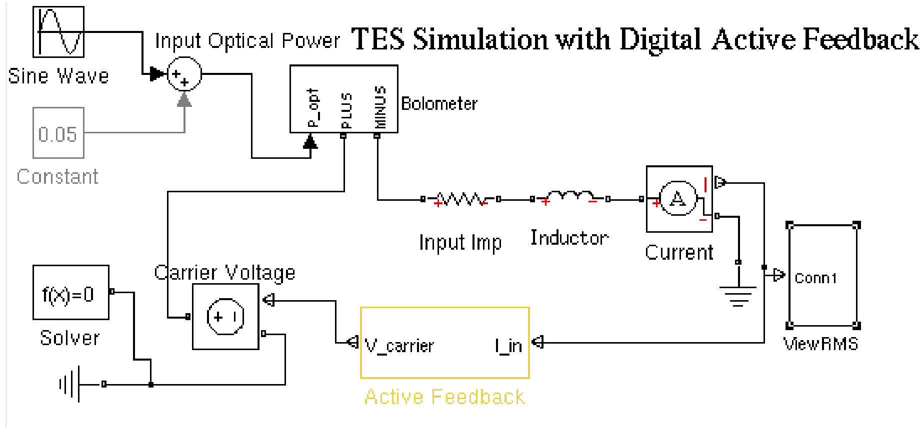

Figure 3: Simulation of a bolometer operating under sinusoidally varying optical power, in series with a readout related complex impedance.

In order to verify the accuracy to which the simulated TES reproduces known characteristic features of voltage biased TES bolometers, I then proceded to varying the voltage across the bolometer and measuring the resulting current. Such an "I-V" curve is shown in Figure 4, accurately reproducing the qualitative features of a TES bolometer.

At low voltage bias, the bolometer is superconducting, leaving only the input impedance of the amplifier as a resistive element. The characteristic $IV = constant$ shows the portion of the IV curve in which the power delivered to the bolometer is a constant: the power required to keep the bolometer at temperature T_c. Deviations from the $IV = constant$ relation indicate instabilities in the bolometer. At high applied voltages the bolometer has a resistance of 1Ω, acting as a purely resistive element.

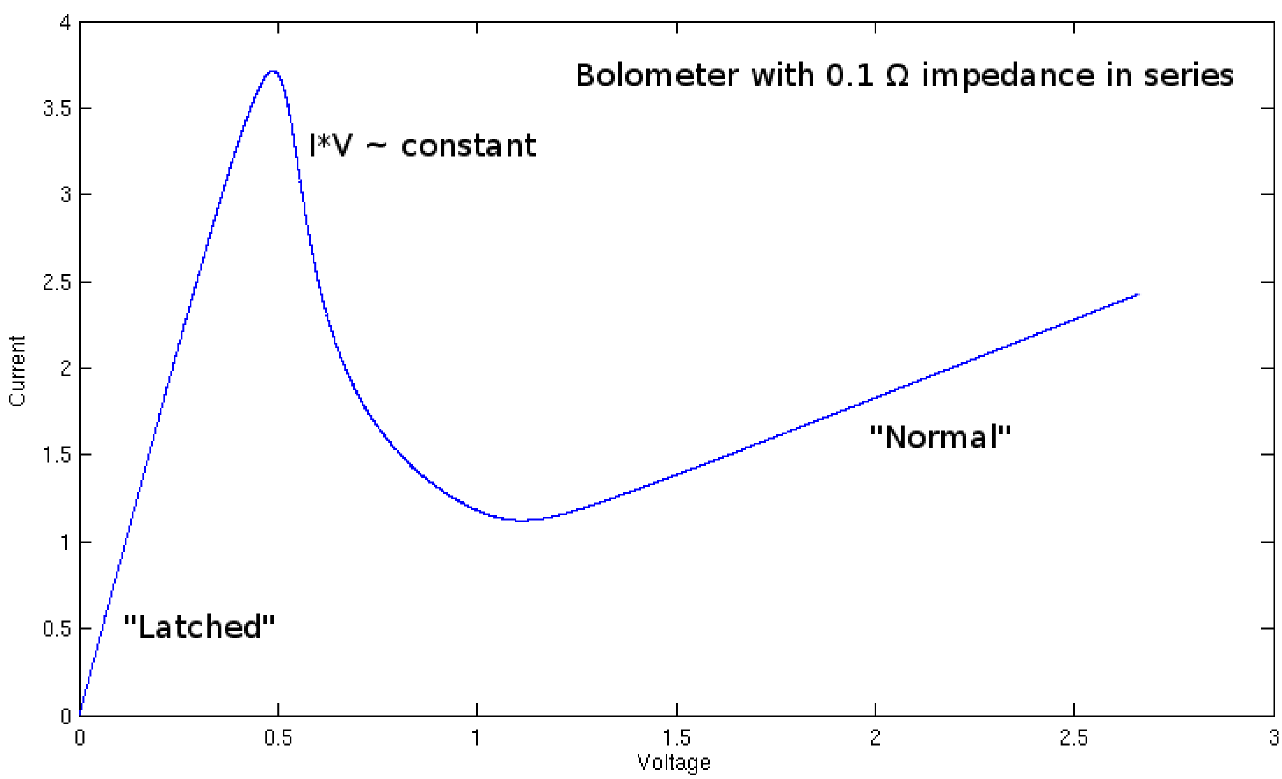

Figure 4: I-V curve for the simulated TES bolometer with a 0.1Ω resistor in series.

3 Active Feedback Simulation

Figure 3 shows a system which takes in the measured current through the bolometer and generates the relevant carrier voltage. Without active feedback, the carrier voltage is generated as shown in Figure 5.

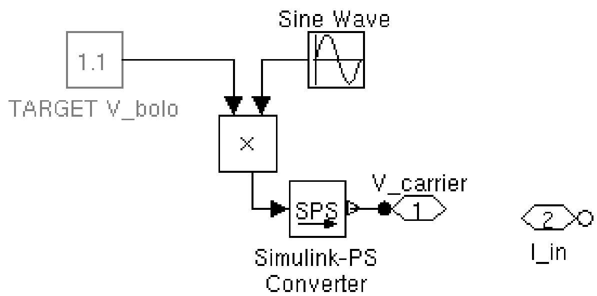

Figure 5: Carrier generation without active feedback.

However, given that the readout system has complex impedance $z_s = R_s + iX_s$ we can use our knowledge of the circuit to write that

$$V_{bolo} = \left[\sqrt{\left(\frac{V_{carrier}}{I}\right)^2 - X_s^2} - R_s \right] I \tag{3}$$

where all variables denote amplitudes. Note that to measure R_s and X_s as a function of frequency, one could latch the bolometer into superconductivity, leaving exactly z_s as the complex impedance of the system.

We can now provide a PID control system to require V_{bolo} to remain constant. When adding this digital active feedback, the simulation is shown in Figure 6.

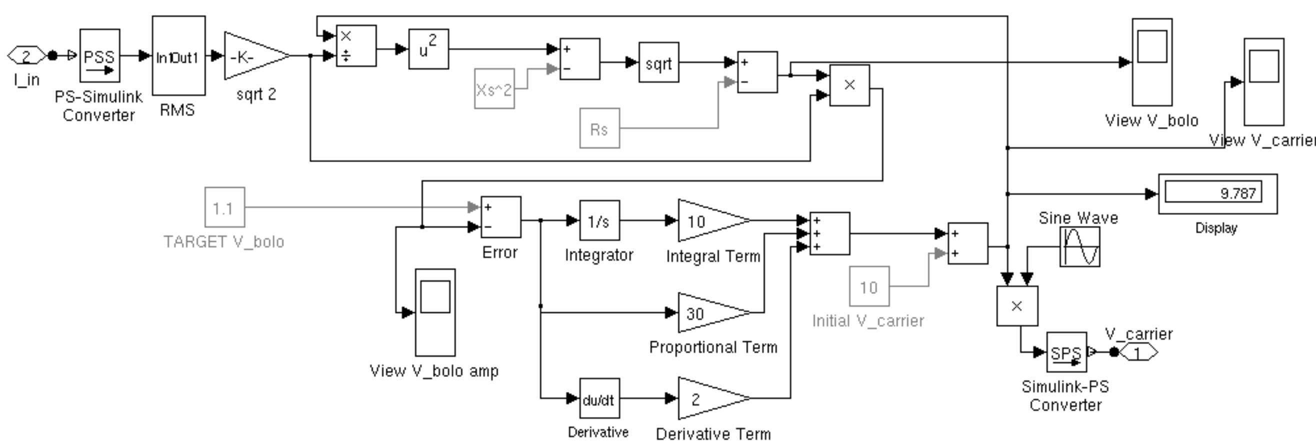

Figure 6: Carrier generation using active feedback in the form of a PID controller.

The three gain parameters of the PID controller can then be chosen to optimize the performance of the active feedback mechanism. Note that we require the condition $\tau_{PID} \ll \tau_{TES}$, in other words the control must occur at shorter timescales than the TES can respond. The amount of PID parameter optimization required depends on how well that condition is met. In my simulation, in order to run in a reasonable amount of time (typically 6 minutes), I satisfied the $\tau_{PID} \ll \tau_{TES}$ condition to a lesser extent than realistically possible in a physical system.

4 Results

I will now demonstrate the performance of active feedback within this simulated system with and without active feedback for two values of amplifier input impedance.

4.1 Without Active Feedback

Figures 7 and 8 show the bolometer response to sinusoidally varying optical power. The bolometer changes its resistance sinusoidally in response to the large changes in applied optical power. Clearly a large input impedance violates the stability condition of a constant voltage bias and the bolometer latches.

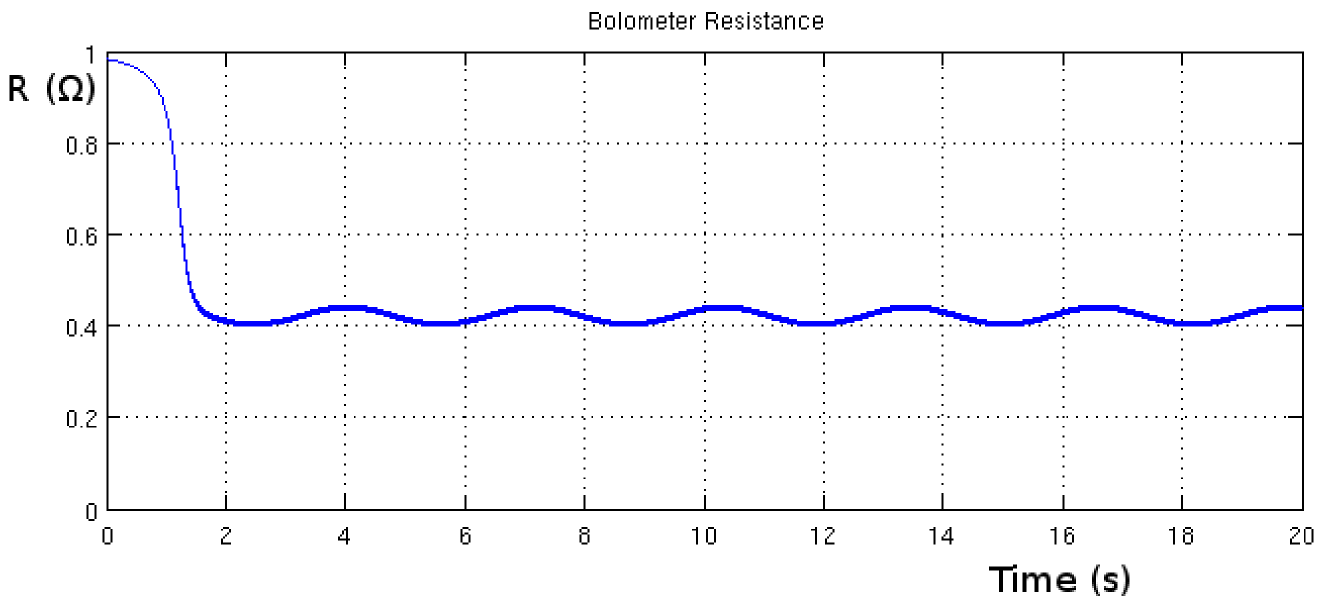

Figure 7: Bolometer resistance while responding to sinusoidal optical power without active feedback with a 0.1Ω amplifier input impedance.

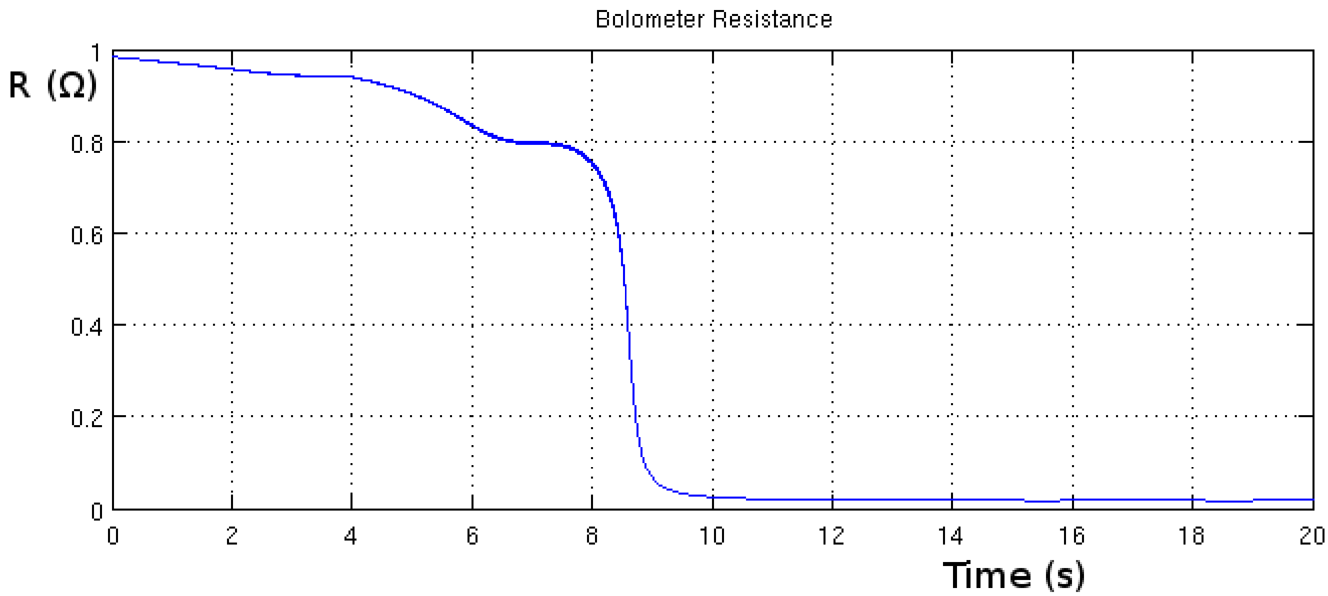

Figure 8: Bolometer resistance while responding to sinusoidal optical power without active feedback with a 2Ω amplifier input impedance. Note that the bolometer is unstable in the sense that it is easily driven normal or superconducting by changes in V_{bias} or optical power.

4.2 With Active Feedback

Figures 9 and 10 show the bolometer response to sinusoidally varying optical power with the digital active feedback mechanism in place. Note that despite the increase in amplifier input impedance, as long as the timescale of the active feedback mechanism is much shorter than that of the bolometer, the stability condition of constant voltage bias is still met and the bolometer functions correctly.

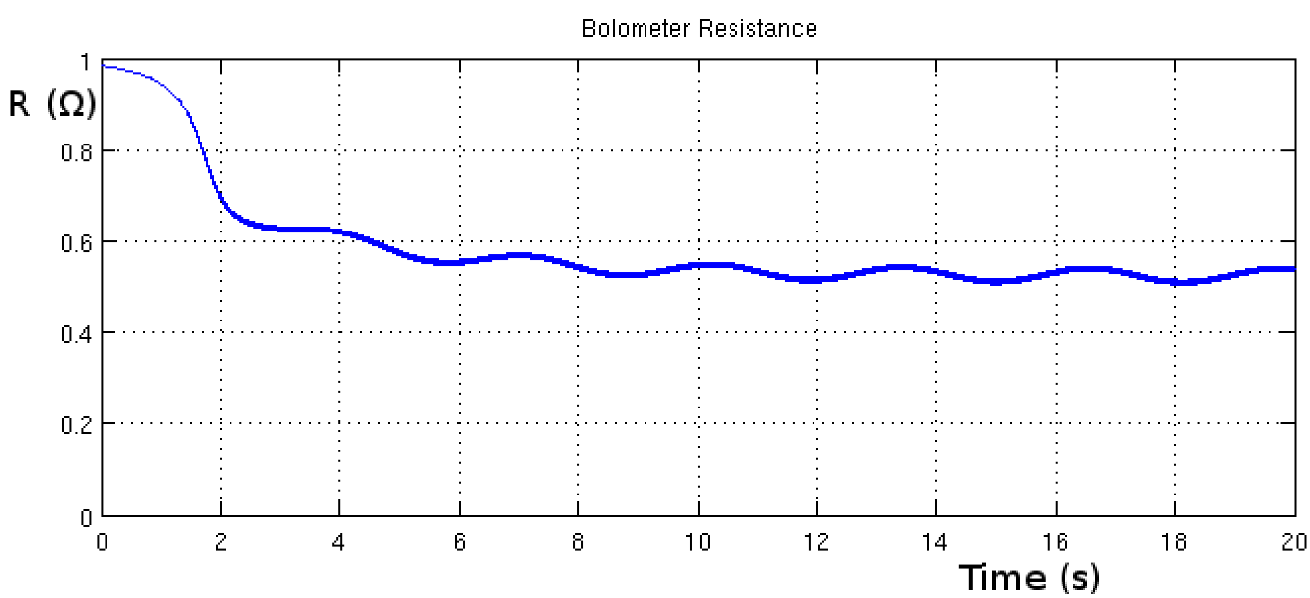

Figure 9: Bolometer resistance while responding to sinusoidal optical power with active feedback with a 0.1Ω amplifier input impedance.

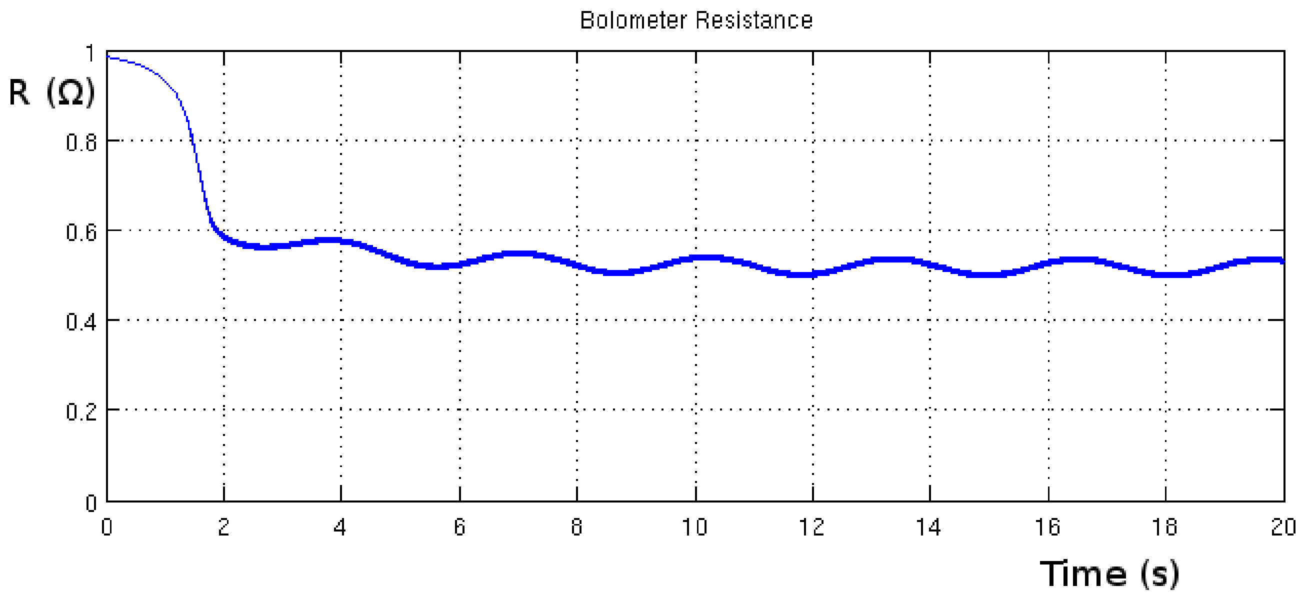

Figure 10: Bolometer resistance while responding to sinusoidal optical power with active feedback with a 2Ω amplifier input impedance.

5 Data Products

Bolometers are used to measure optical power. In existing systems, the current measured by the amplifier is often downsampled to ~ 100 Hz and stored to disk. The resulting data product is then proportional to the measured sky power, the relevant proportionality constant being the bias voltage. In a system with digital active feedback, the measured current should stil be proportional to the incident optical power, since V_{bolo} should still be constant. Simulations show that a significant misestimation of any one of the parameters I, X_s or R_s results in an erroneous measurement of V_{bolo}. However, this misestimation is only quadratic in the error on the aforementioned parameters. Therefore, storing I and V_{bolo} to disk separately may be advisable in the initial development of the active feedback circuit.

6 Noise

Any noise manifesting as "resistance noise", such as shot noise from incident photons or phonon noise on the bolometer, are rolled off by the TES time constant. Therefore, the noise properties are unchanged under active feedback.

In contrast, Johnson noise on the TES can be interpreted as voltage noise. Therefore only noise near the carrier frequency (to which the demodulator is sensitive) is relevant. I will now show that Johnson noise also manifests as "resistance noise" and hence is unaffected by active feedback.

Consider a single frequency mode of Johnson voltage noise such that the effective carrier voltage is

$$V = V_b \sin[\omega t] + \epsilon V_b \sin[\lambda t] \tag{4}$$

Johnson noise can be considered a sum of these types of signals. The power on the bolometer then becomes

$$P = \frac{V^2}{R} = \frac{V_b^2 \sin^2[\omega t] + 2V_b^2 \epsilon \sin[\omega t]\sin[\lambda t]}{R} + \mathcal{O}(\epsilon^2) \tag{5}$$

Since the TES only changes in resistance due to applied electrical power on long timescales, we can ignore the various quickly varying terms and obtain

$$P = \frac{\frac{V_b^2}{2} + \epsilon V_b^2 \cos\left[(\omega - \lambda)t\right]}{R} + \mathcal{O}(\epsilon^2) \tag{6}$$

Hence, the resistance of the TES becomes

$$R \approx R_0 + \epsilon V_b^2 \frac{\mathrm{d}R}{\mathrm{d}P} \frac{\cos\left[(\omega - \lambda)t\right]}{R_0} + \mathcal{O}(\epsilon^2) \tag{7}$$

where $\frac{\mathrm{d}R}{\mathrm{d}P}$ is the responsivity of the bolometer. The measured current then becomes

$$I \approx V_b \frac{\sin\left[\omega t\right] + \epsilon \sin\left[\lambda t\right]}{R_0} - \epsilon V_b^2 \frac{\mathrm{d}R}{\mathrm{d}P} \frac{\sin\left[(2\omega - \lambda)t\right] + \sin\left[\lambda t\right]}{2R_0^3} + \mathcal{O}(\epsilon^2) \tag{8}$$

where I have kept only the terms near the carrier frequency. Effectively, this shows (to order ϵ^2) that the strong electrothermal feedback creates sidebands on the carrier near the carrier frequency. Because these sidebands are out of phase with the additional injected signal term, they interfere and suppress the signal in a manner analogous to Mathers (1982).

Note that the Johnson noise is suppressed by a factor of

$$\frac{\sqrt{\left(\epsilon\left(1 - \frac{\mathrm{d}R}{\mathrm{d}P_e}\frac{V_b^2}{2R_0}\right)\right)^2 + (\epsilon)^2}}{\epsilon} \tag{9}$$

This is because the newly generated sideband adds in quadrature with the amplitude reduced sideband to form the reduced total noise amplitude. Note that P_e is the electrical power applied to the bolometer assuming it is purely resistive.

Given that $\delta P_e = -\mathcal{L}\delta P$ we can rewrite

$$\frac{\mathrm{d}R}{\mathrm{d}P_e} = -\frac{1}{\mathcal{L}}\frac{\mathrm{d}R}{\mathrm{d}T}\left(\frac{\mathrm{d}P}{\mathrm{d}T}\right)^{-1} = -\frac{R_0\alpha}{T_c}(G)^{-1} \tag{10}$$

But Equation (3.10) in Trevor's thesis shows that $\mathcal{L} = P_e\alpha/GT_c$ so that we can cancel factors and upon substituting into Equation 9 obtain that the Johnson noise reduction is exactly $1/\sqrt{2}$.

Since the Johnson noise acts effectively on the bolometer resistance (just as the other types of noise discussed above) the active feedback should have no effect on the signal to noise ratio.

7 Stability

The previous sections assumed that the RMS current in the system could be measured. However, a lock-in demodulator typically locks into the phase of the current and measures only that component. If we define the carrier voltage to be

$$V_{carrier} = |V|\mathrm{e}^{i\omega t} \tag{11}$$

then we can define the current in the system to be

$$I = |I|\mathrm{e}^{i\omega t + \phi} \tag{12}$$

where

$$\phi = \arctan\left[\frac{X_s}{R_s + R_{bolo}}\right] \tag{13}$$

This phase shift can be "locked onto" by the demodulator. However, if we perturb the phase $\phi \rightarrow \phi + \delta\phi$ then our measured current will be affected. Hence, the estimate of V_{bolo} in the control loop will be erroneous.

Under the perturbation $\phi \rightarrow \phi + \delta\phi$, the effective result of the control loop is $R_{bolo} \rightarrow R_{bolo} + \delta R_{bolo}$. Logically following the action of the control system shows that $\delta R_{bolo} < 0$, regardless of the sign of $\delta\phi$.

The perturbation in the current is

$$\delta I = \frac{V_{bolo}}{R_{bolo} + R_s + iX_s} - \frac{V_{bolo}}{R_{bolo} + \delta R_{bolo} + R_s + iX_s} \tag{14}$$

combining terms yields

$$\delta I = \frac{V_{bolo} \delta R_{bolo} \left(\left[X_s^2 - (R_{bolo} + R_s)^2 - \delta R_{bolo} \right] + i \left[X_s \left(2R + \delta R_{bolo} \right) \right] \right)}{\left((R_{bolo} + R_s + \delta R_{bolo})^2 + X_s^2 \right) (R^2 + X_s^2)} \tag{15}$$

However, we are only sensitive to one component as shown in Figure 11. If we let

$$\mathbf{u}(z) = \begin{bmatrix} \mathbb{R}(z) \\ \mathbb{I}(z) \end{bmatrix} \tag{16}$$

where z is some complex number then we can project out the relevant component by writing

$$\mathbf{u}(I_{||}) = \mathbf{u} \left(\frac{I}{|I|} \right) \mathbf{u}^{\mathrm{T}} \left(\frac{I}{|I|} \right) \mathbf{u}(\delta I) \tag{17}$$

which, after some algebra, becomes (along the direction of I)

$$\delta I = \frac{V_{bolo} \delta R_{bolo} \left(R_s + R_{bolo} \right) \left((R_s + R_{bolo})^2 - X_s^2 \right)}{\left((R_s + R_{bolo})^2 + X_s^2 \right)^2} \tag{18}$$

where I have ignored terms of order δR_{bolo}^2. Note that the effect of an instantaneous phase perturbation $\delta\phi$ is an instantaneous decrease in the measured I. If the active feedback then provides $\delta I > 0$, the system *must* be stable.

Using Equation 18, this shows that $R_s + R_{bolo} > X_s$ guarantees stability. In other words when the phase of the current is within 45° of the phase of the carrier voltage, stability is guaranteed. Simulations show that even when $X_s > R_s + R_{bolo}$, stability can still be achieved under certain PID parameters and/or optical signals.

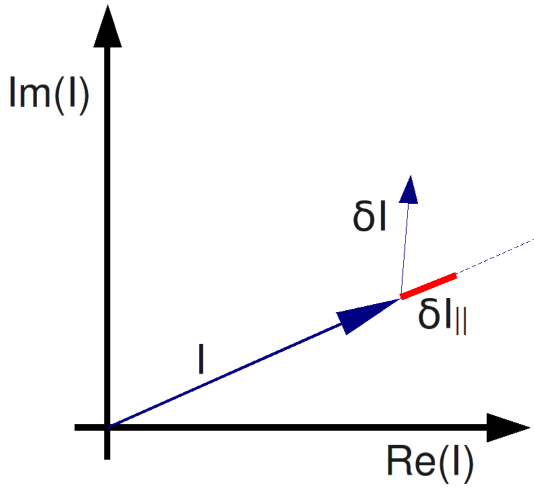

Figure 11: The positivity of the component of δI shown in red is the relevant stability condition.

8 Conclusion

Provided it occurs on fast timescales, the active feedback mechanism allows for a constant voltage bias to the bolometer, despite violation of the $z_{amp} \ll z_{bolo}$ condition.