Effect of series impedance to bolometer responsivity

Tijmen de Haan

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This quick memo is one of my usual small signal expansions of the bolometer power balance equation. Specifically, I calculate "excess responsivity" due to impedance in series with the bolometer. Let's start with the power balance equation:

$$P_{opt} + \frac{V_b^2}{R_b} = G\Delta T \tag{1}$$

which contains a static part which says that the steady state optical loading plus electrical power equals the power flowing off the bolometer through G. To perform a small signal analysis, we equate the first order terms of the Taylor expansion of Equation 1

$$\delta P_{opt} + \frac{2V_b \delta V_b}{R_b} = \frac{V_b^2}{R_b^2}\delta R_b \frac{1+\mathcal{L}}{\mathcal{L}} \tag{2}$$

See http://kingspeak.physics.mcgill.ca/twiki/bin/edit/ColdFeedback/LoopGainEquations for a detailed explanation of the loop gain $\mathcal{L}$ and a more thorough derivation of this result.

I denote stray impedance in series with the bolometer as the (in general complex) quantity z_s. It relates the total carrier voltage V_c to the voltage drop across the bolometer V_b through

$$V_b = \frac{V_c R_b}{R_b + z_s} \tag{3}$$

Substituting this in and rearranging Equation 2 gives

$$\delta P_{opt} = \delta R_b \left(\frac{V_c^2 \mathcal{L}^{-1}}{R_b^{(0)}(R_b^{(0)} + z_s)} + \frac{2V_c^2 R_b^{(0)}}{(R_b^{(0)} + z_s)^3} - \frac{V_c^2}{(R_b^{(0)} + z_s)^2} \right) \tag{4}$$

The current through the bolometer is

$$I = \frac{V_c}{R_b + z_s} \tag{5}$$

which we can expand to first order as

$$\delta I = -\frac{V_c + \delta R_b}{(R_b^{(0)} + z_s)^2} \tag{6}$$

We can then solve for the responsivity

$$S \equiv \frac{\delta I}{\delta P_{opt}} = -\frac{1}{V_c}\left(\frac{R_b^{(0)} + z_s}{\mathcal{L} + R_b^{(0)}} + \frac{2R_b^{(0)}}{R_b^{(0)} + z_s} - 1 \right)^{-1} \tag{7}$$

Expanding in $z_s/R_b^{(0)}$ i.e. the fractional stray gives the following.

$$S = -\frac{1}{V_c}\frac{\mathcal{L}}{\mathcal{L}+1}\left(1 - \frac{z_s}{R_b^{(0)}}\frac{1-2\mathcal{L}}{1+\mathcal{L}}\right) \tag{8}$$

Finally, in the limit of high loopgain

$$\lim_{\mathcal{L}\to\infty} S = -\frac{1}{V_c}\left(1 + \frac{2z_s}{R_b^{(0)}}\right) \tag{9}$$

so, for instance, if we have 0.15 Ω of real impedance in series with a 1 Ω bolometer, we'd see a 30% increase in responsivity.